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$$x(t + \Delta t) = x(t) + \dot{x}\,\Delta t \quad + O(\Delta t^2)$$
$$\dot{x}(t + \Delta t) = \dot{x}(t) + a(t)\,\Delta t \quad + O(\Delta t^2)$$

We say “first order solver” since errors are on order of Δt^2

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4th order solver

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4th order solver

$$\boldsymbol{x}_p = \boldsymbol{x}_0 + \dot{\boldsymbol{x}}_0\Delta t + \tfrac{1}{2}\boldsymbol{a}_0\Delta t^2 + \tfrac{1}{6}\dot{\boldsymbol{a}}_0\Delta t^3$$
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Need “predictive” step for a and “jerk” $\dot{a}$ otherwise this is circular (need x_1 for a_1, but need a_1 for x_1…)

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See Resources section for more information about integrators.

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One more thing: “Gravitational Units”

Basically: G = M = R = 1

N-body Units

A conventional system of units in which

$$\begin{aligned} G &= 1 \\ M &= 1 \\ R &= 1 \end{aligned}$$

Example

Suppose a star cluster has $M = 10^5 M_\odot, R = 5$pc. To convert a velocity from the N-body code to km/s, multiply by $\sqrt{\frac{GM}{R}}$, where G is expressed in the same units (i.e. km/s, $M_\odot$, pc), i.e. $G \simeq 0.043$.

(slides brazenly stolen from Heggie)

On day 3 website:
Some code that compares Hermite and Euler Integration Schemes with Analytical

www.astroblend.com/ba2017/code/day3/twobodyintegrators_eu_and_hermite_jills.py