A Week of Computational Astronomy

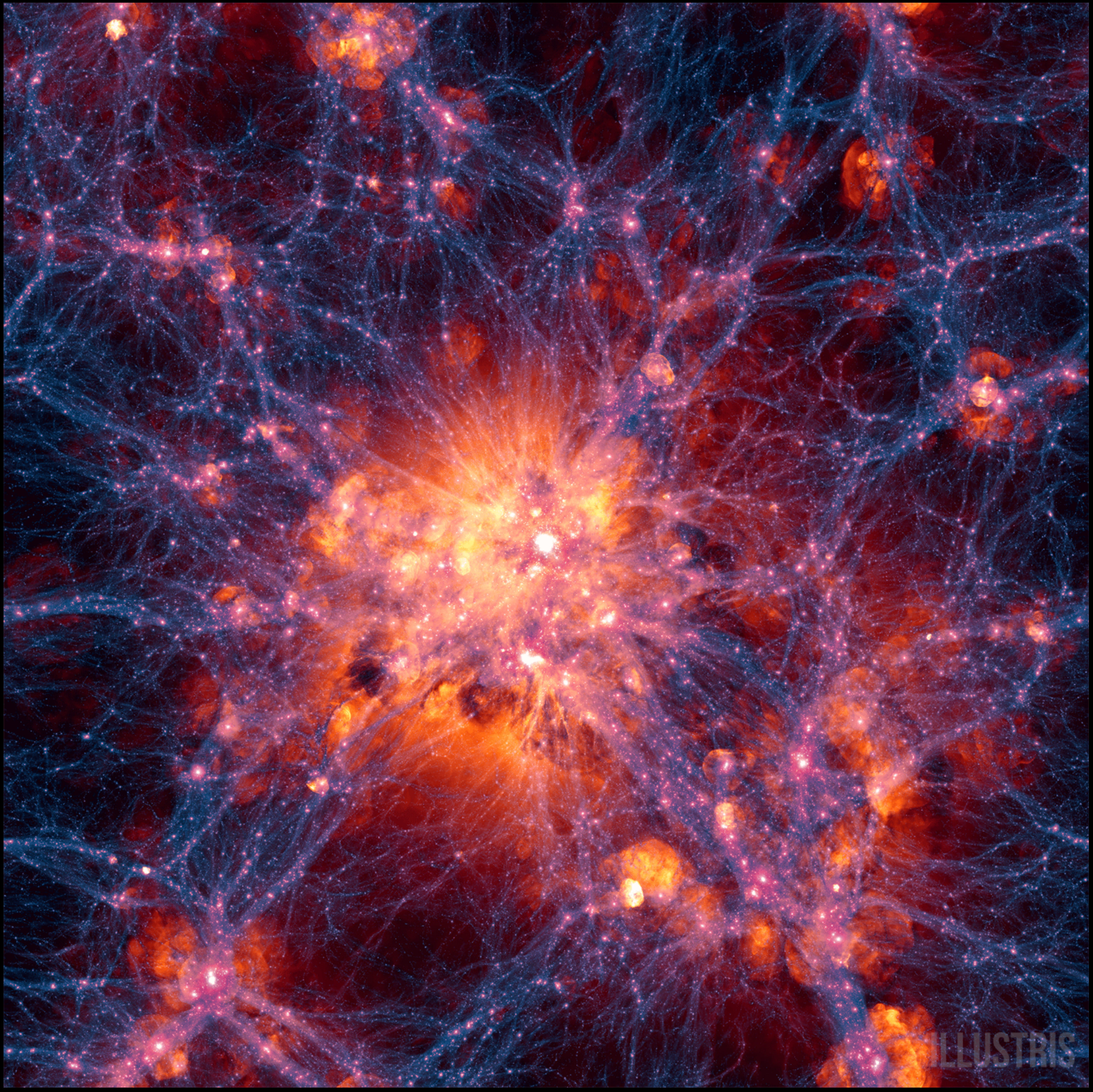
www.astroblend.com/ba2016

A Week of Computational Astronomy

… beginning with some motivation
(aka super cool movies and pictures)

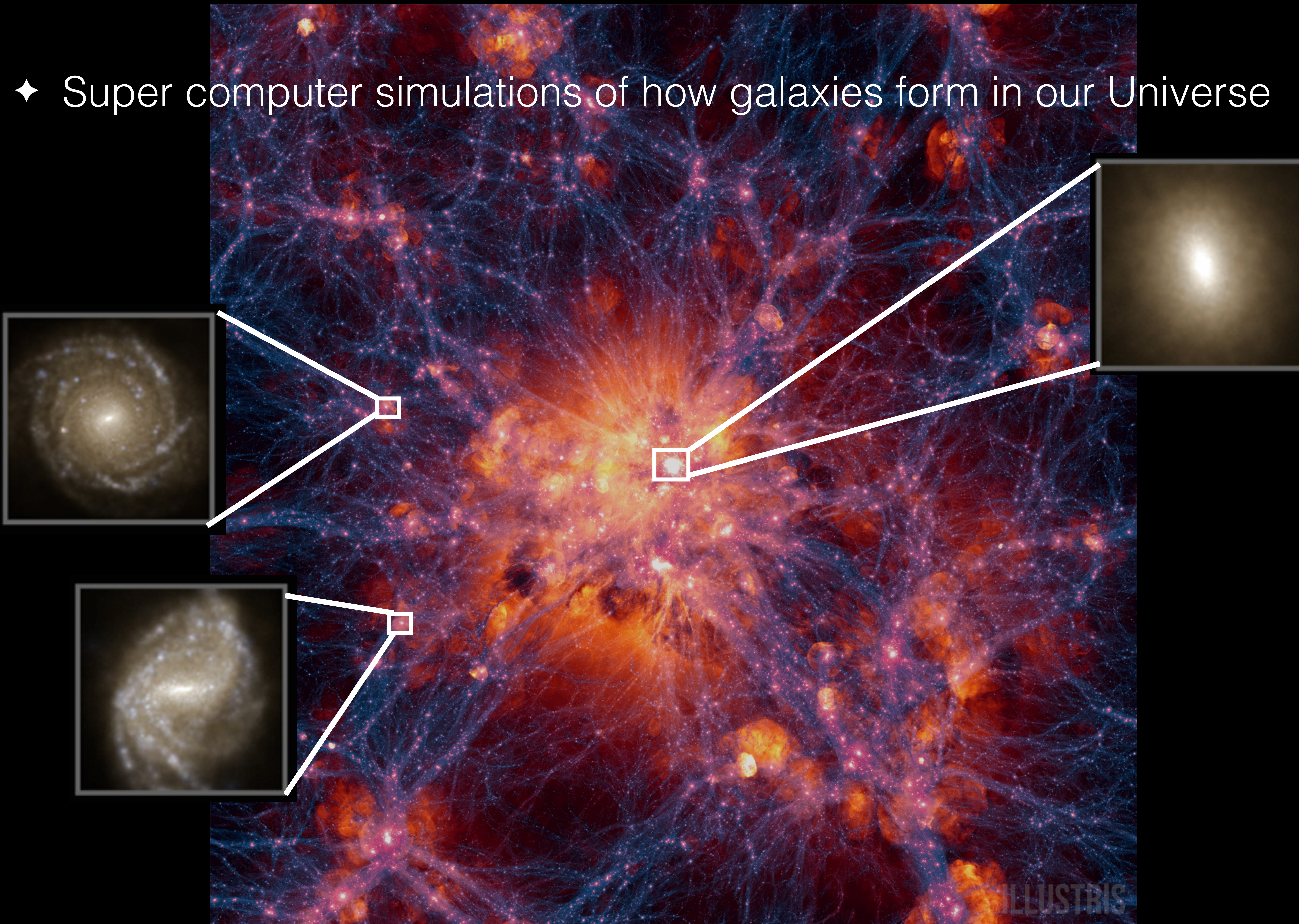
(1) Who am I?
(2) What are we doing?
(3) How are we gonna do it?

Who are you and what do you do?



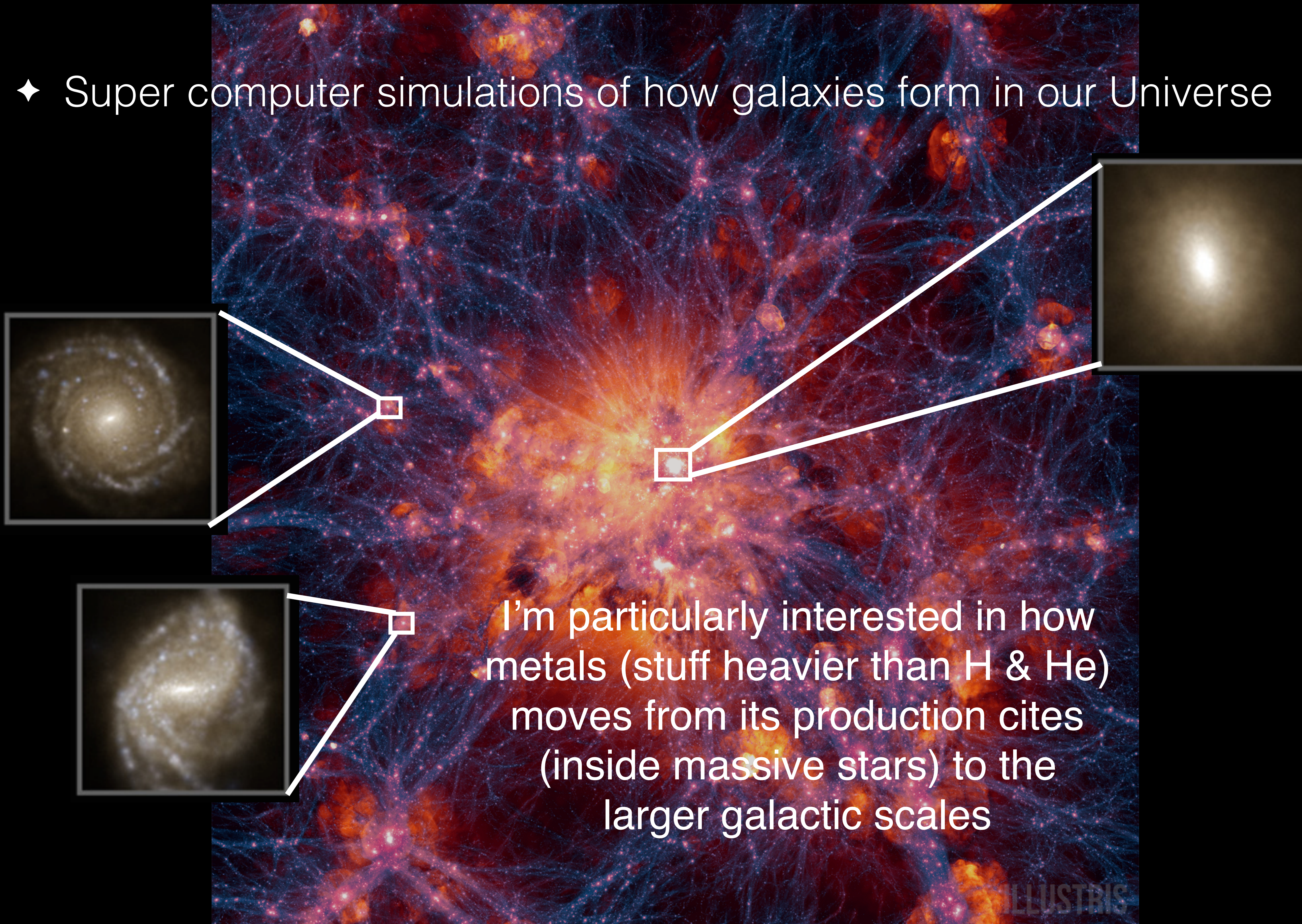
Who are you and what do you do?

✦ Super computer simulations of how galaxies form in our Universe



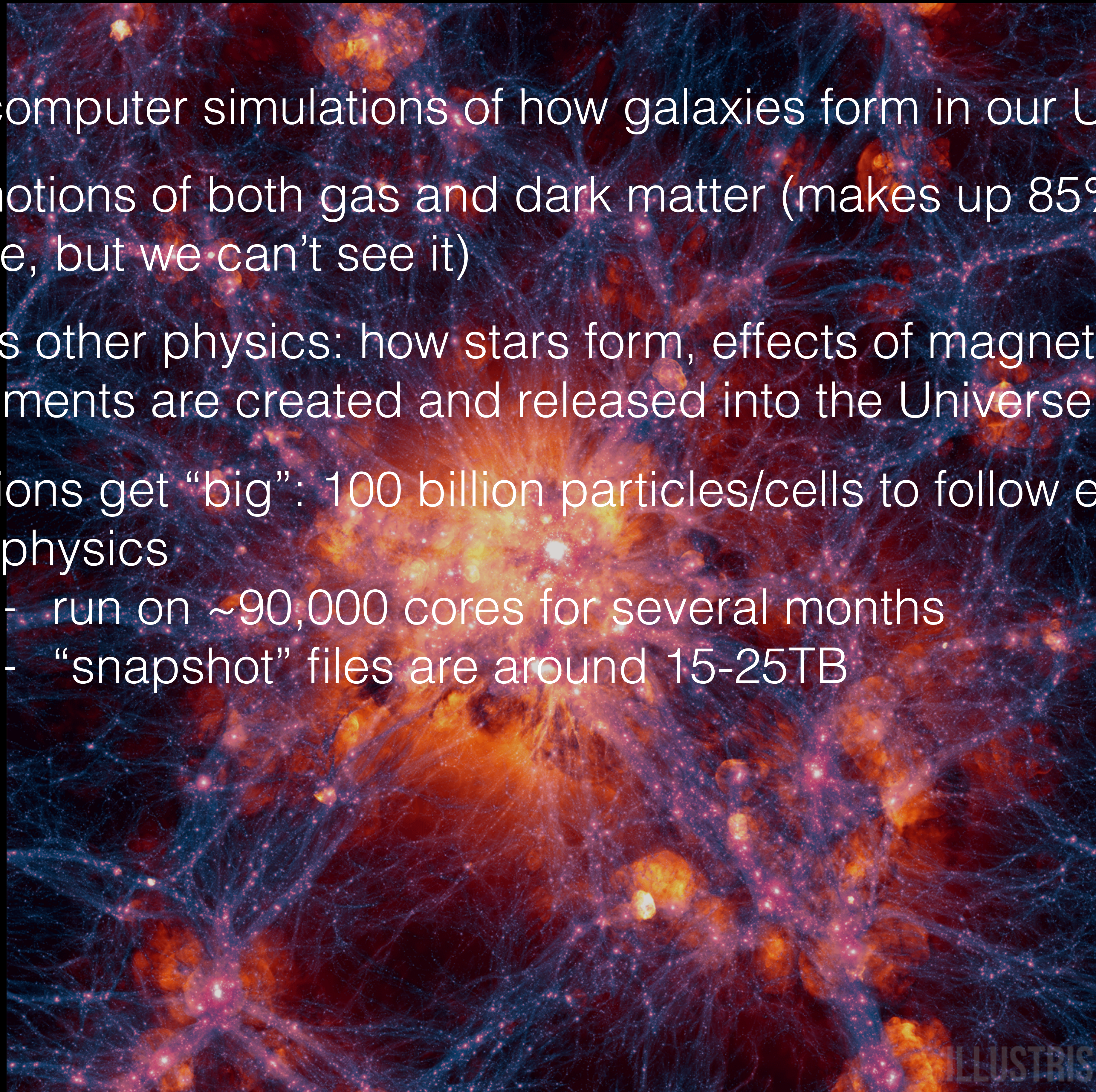
Who are you and what do you do?

- Super computer simulations of how galaxies form in our Universe



I'm particularly interested in how metals (stuff heavier than H & He) moves from its production cites (inside massive stars) to the larger galactic scales

Who are you and what do you do?



- Super computer simulations of how galaxies form in our Universe
- Track motions of both gas and dark matter (makes up 85% of the Universe, but we can't see it)
- Includes other physics: how stars form, effects of magnetic fields, how elements are created and released into the Universe, etc
- Simulations get "big": 100 billion particles/cells to follow each with its own physics
    - run on ~90,000 cores for several months
    - "snapshot" files are around 15-25TB

And now… some gratuitous movies!

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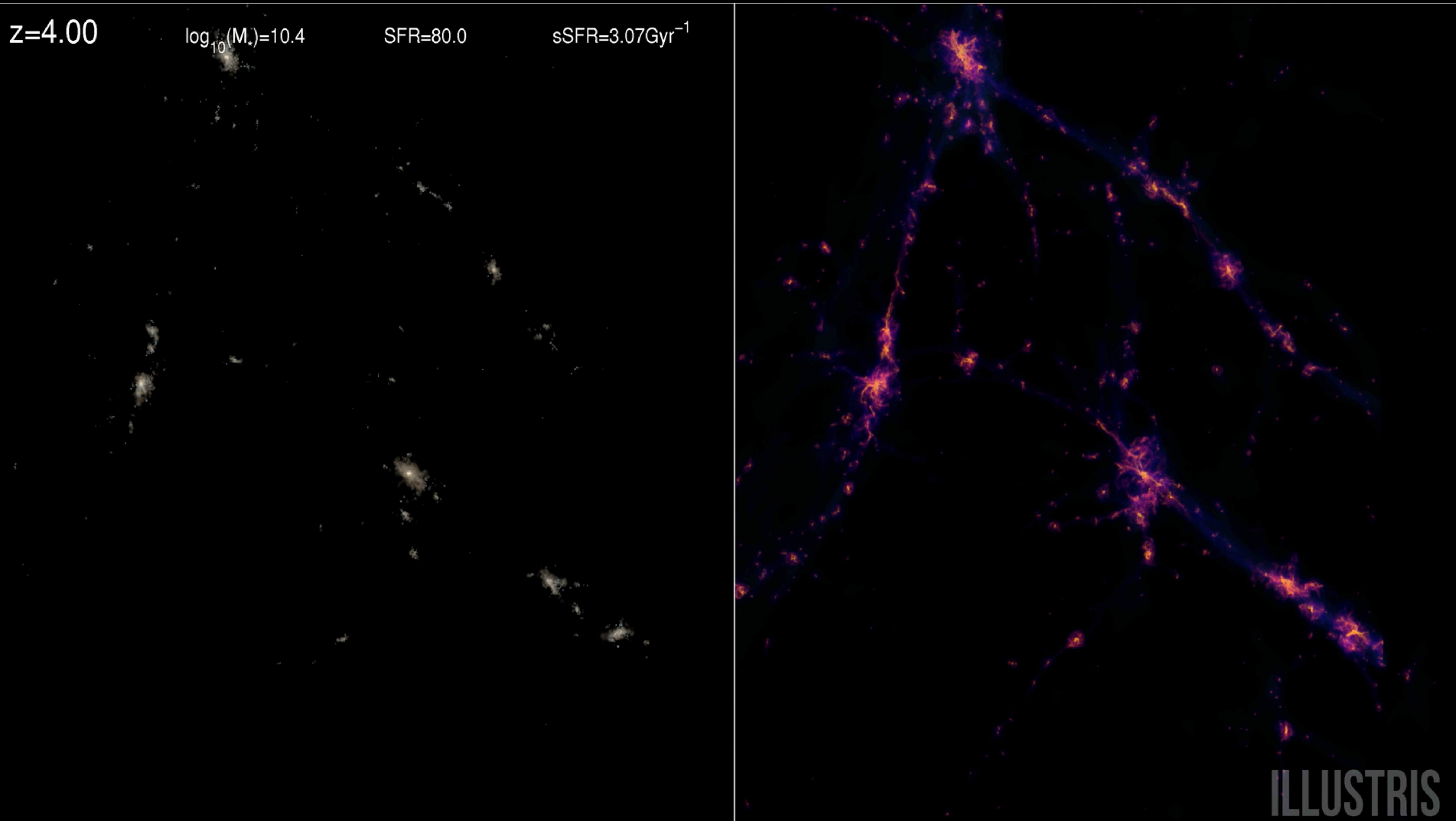

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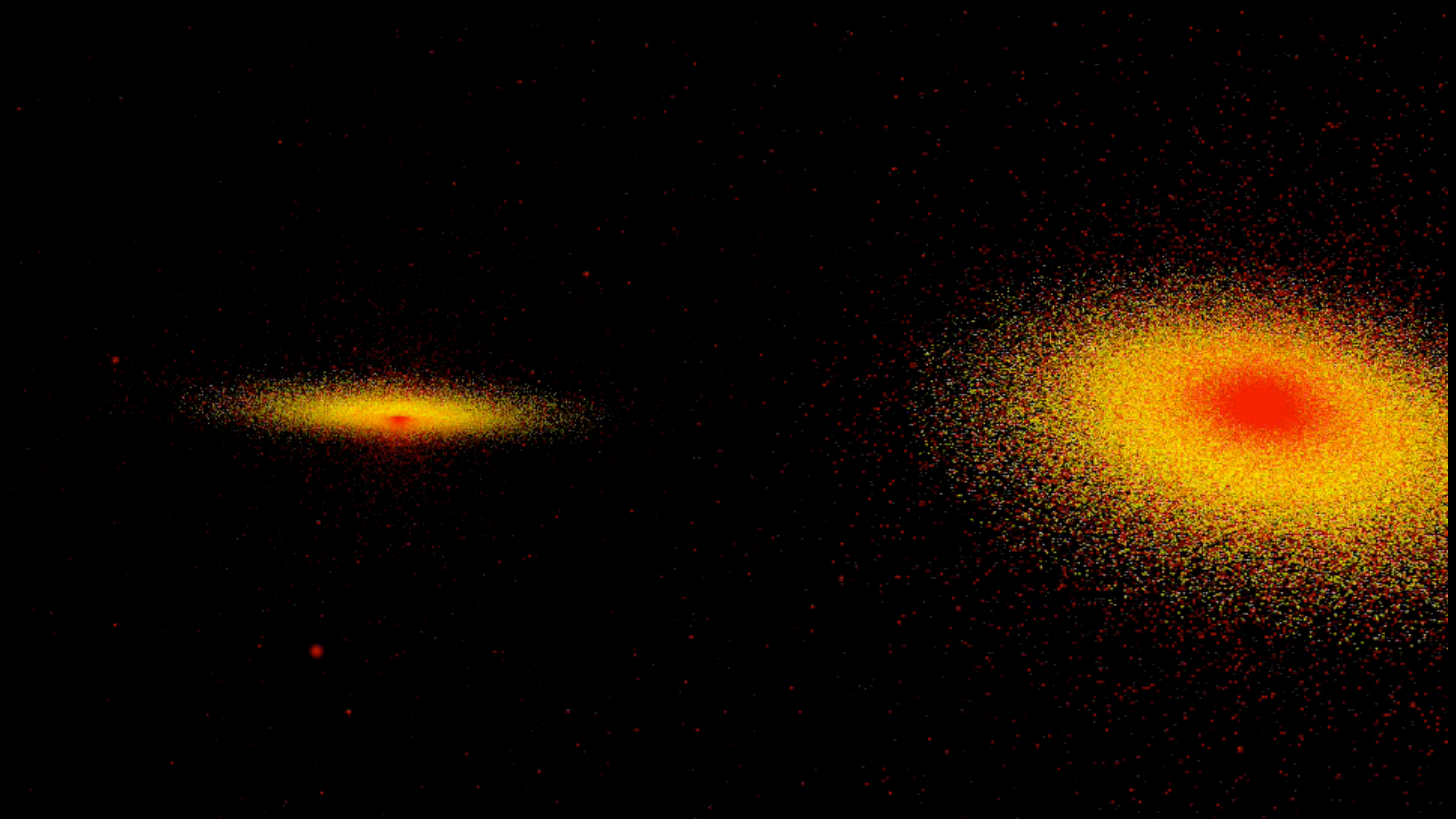


~4 billion particles

… Galaxy Mergers are an N-Body problem

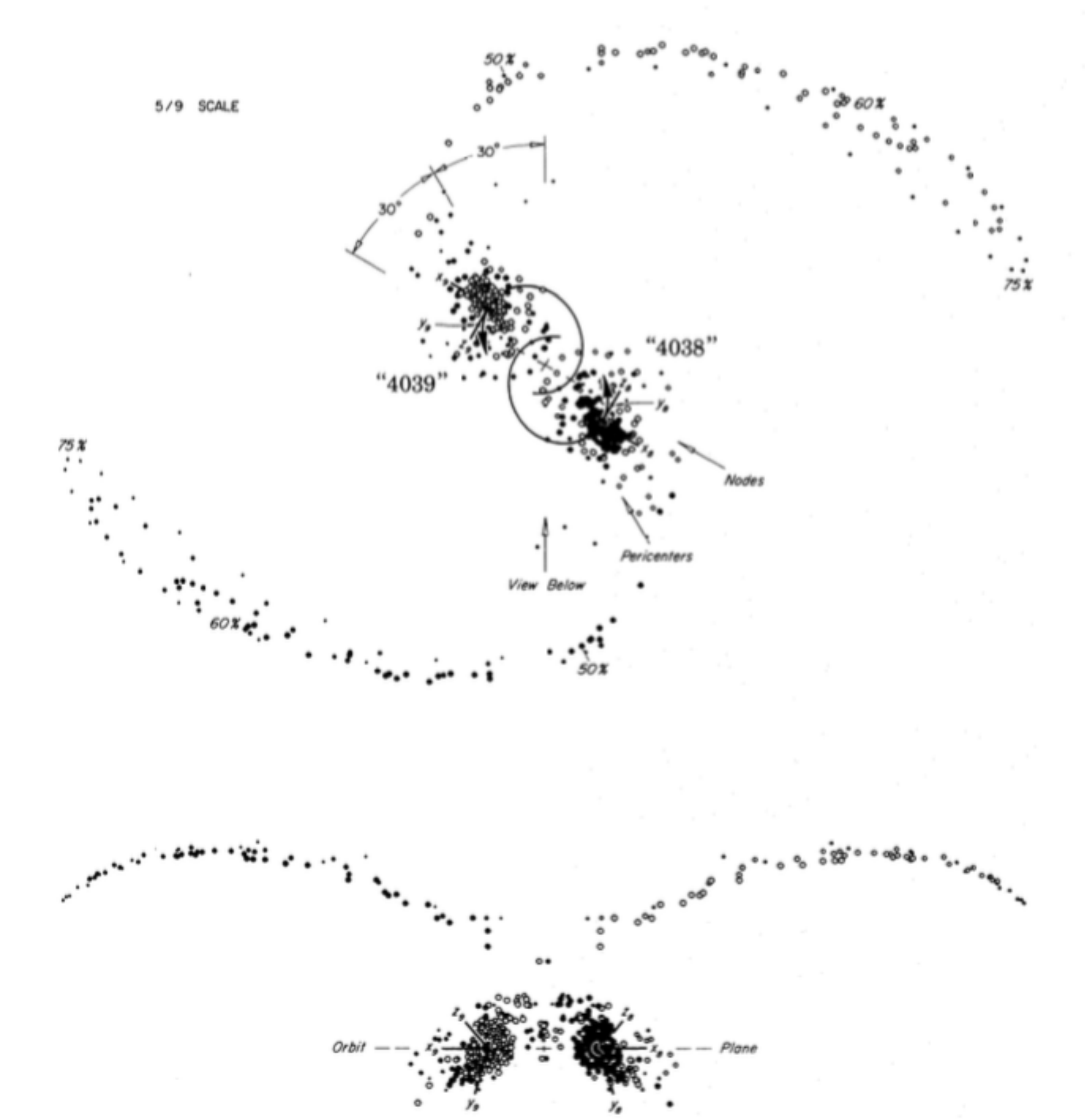
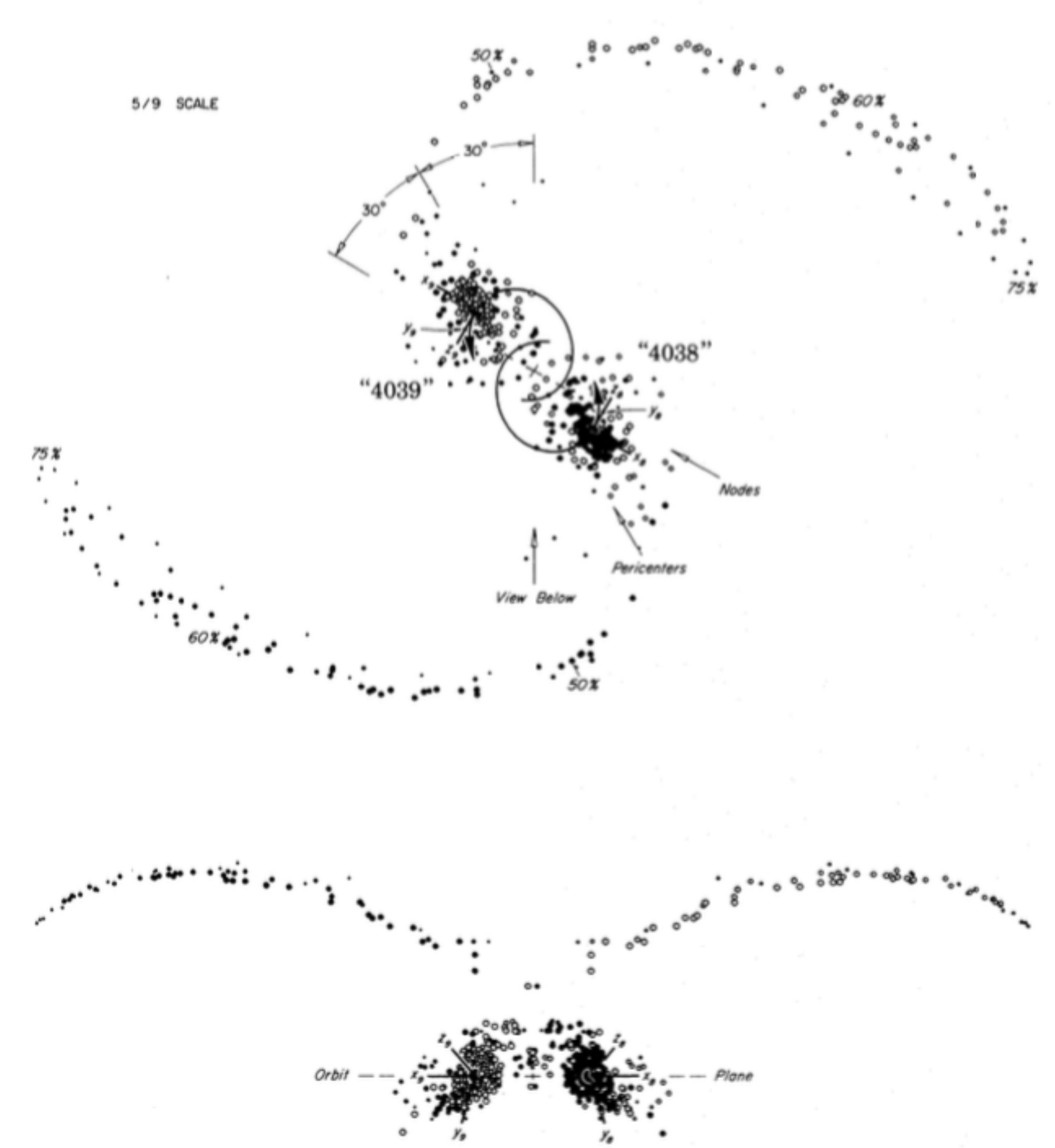


FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{\min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

Toomre & Toomre in the 1970's wanted to explain the observed tidal tails and bridges between galaxies - thought it might have to do with mergers.

… Galaxy Mergers are an N-Body problem

~300 particles!



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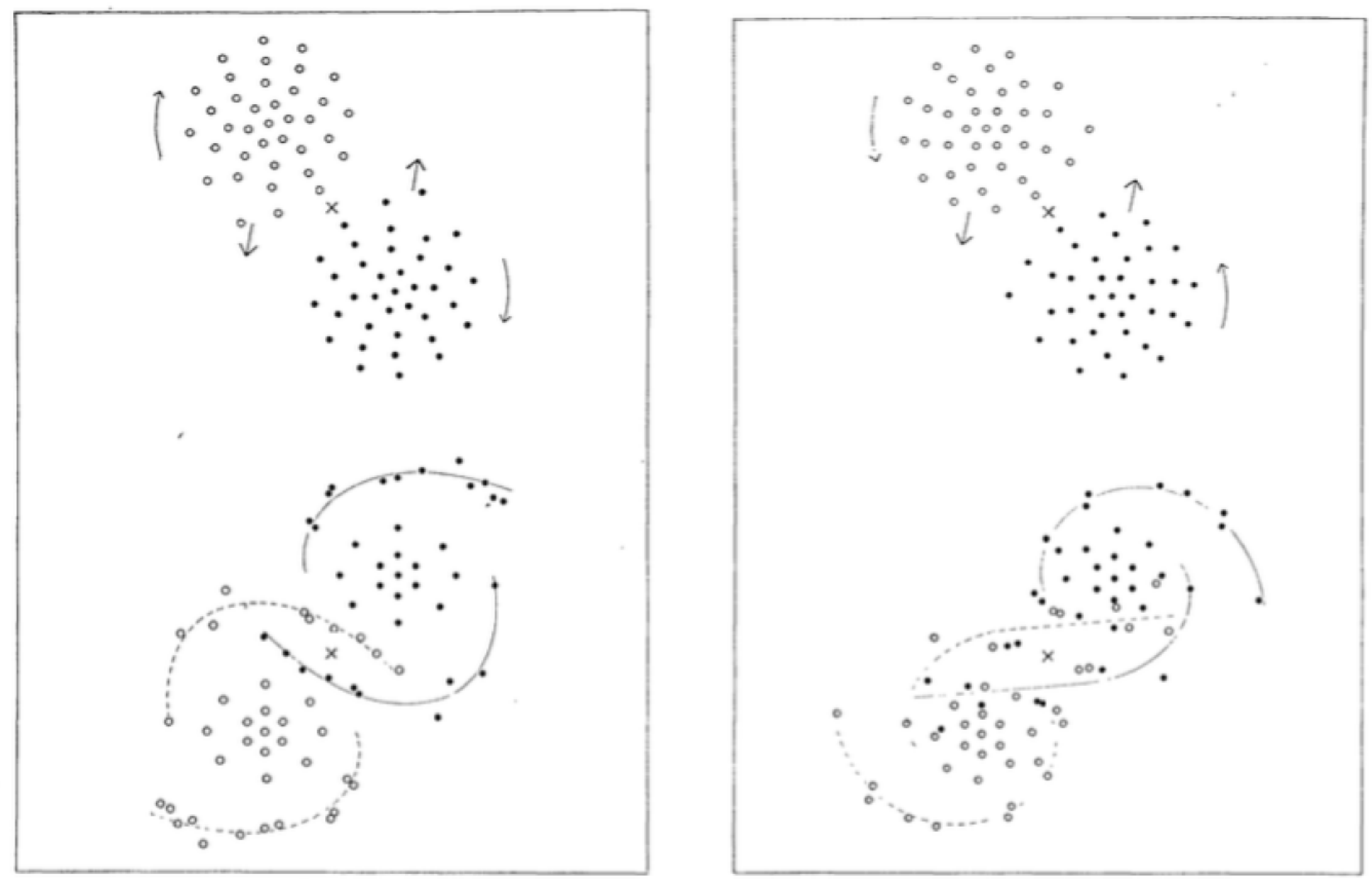


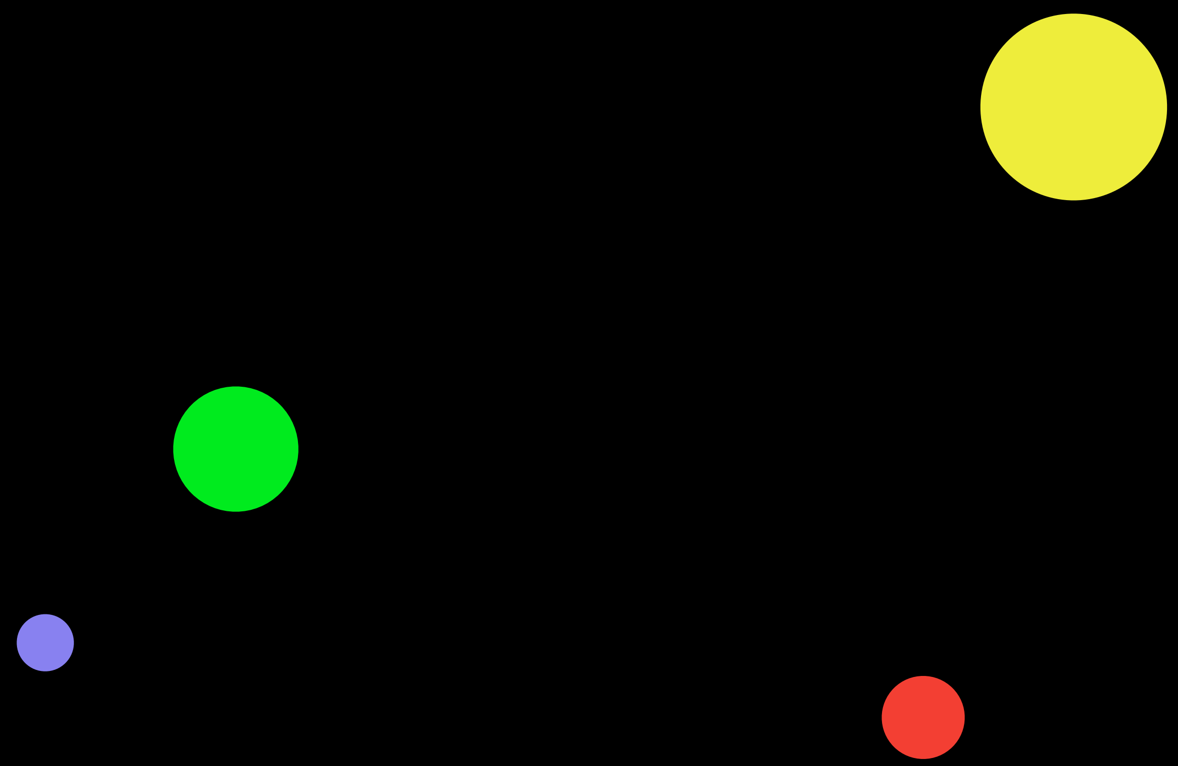
Fig. 4a Fig. 4b

Fig. 4a.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

Fig. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

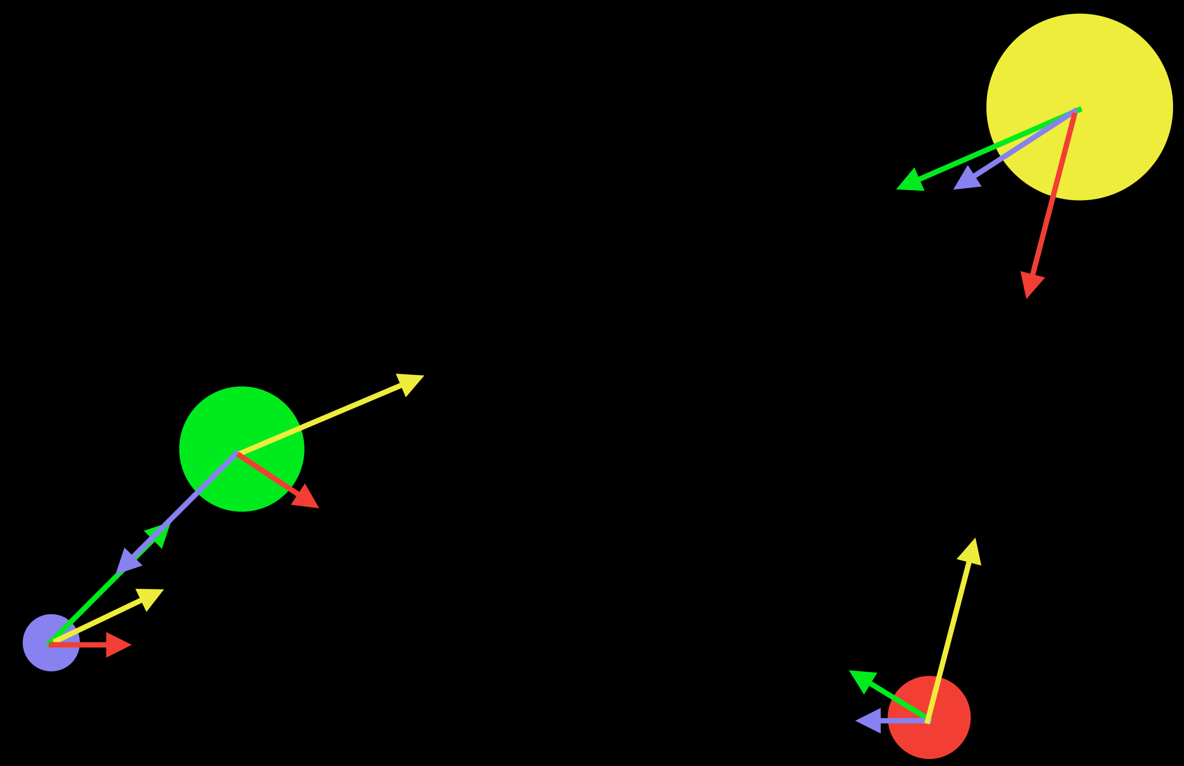
Cool bit of history: Holmberg 1941 did the N-body problem using overlapping lights and photocells to figure out density and backtrack out gravitational force between particles.

N-Body Simulations



N-Body Simulations

For each particle you have to calculate the force exerted on it by each other particle



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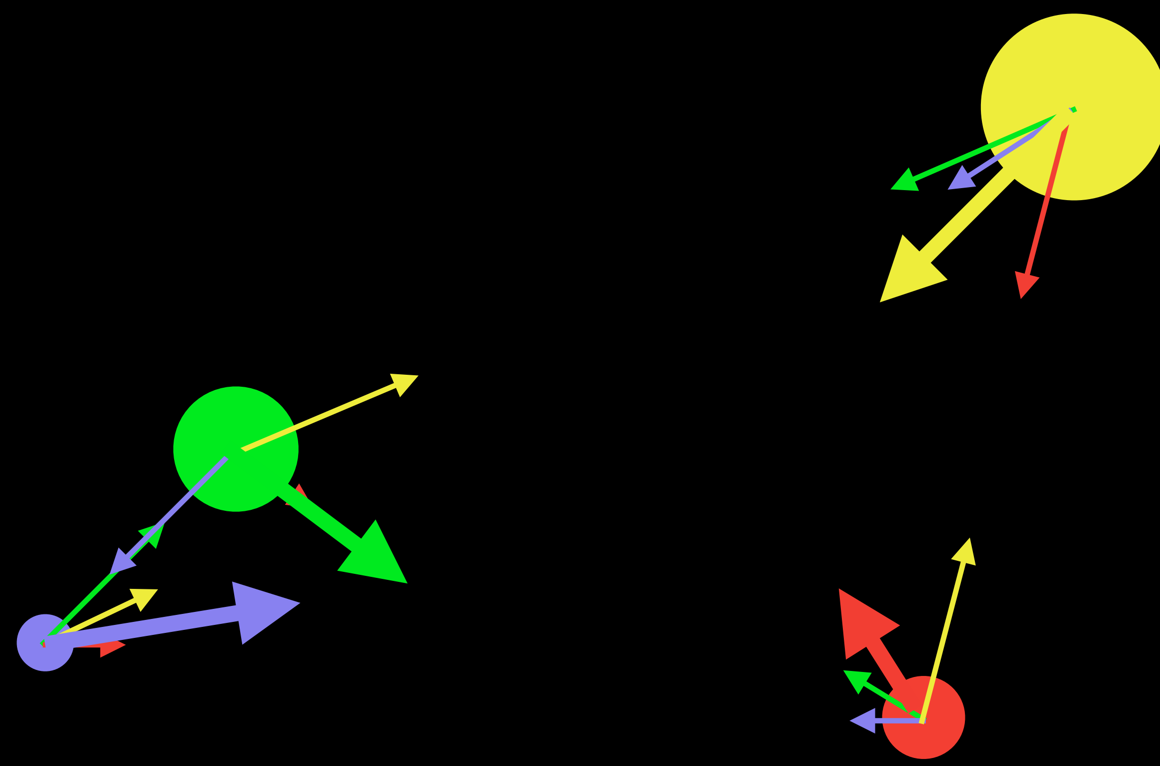

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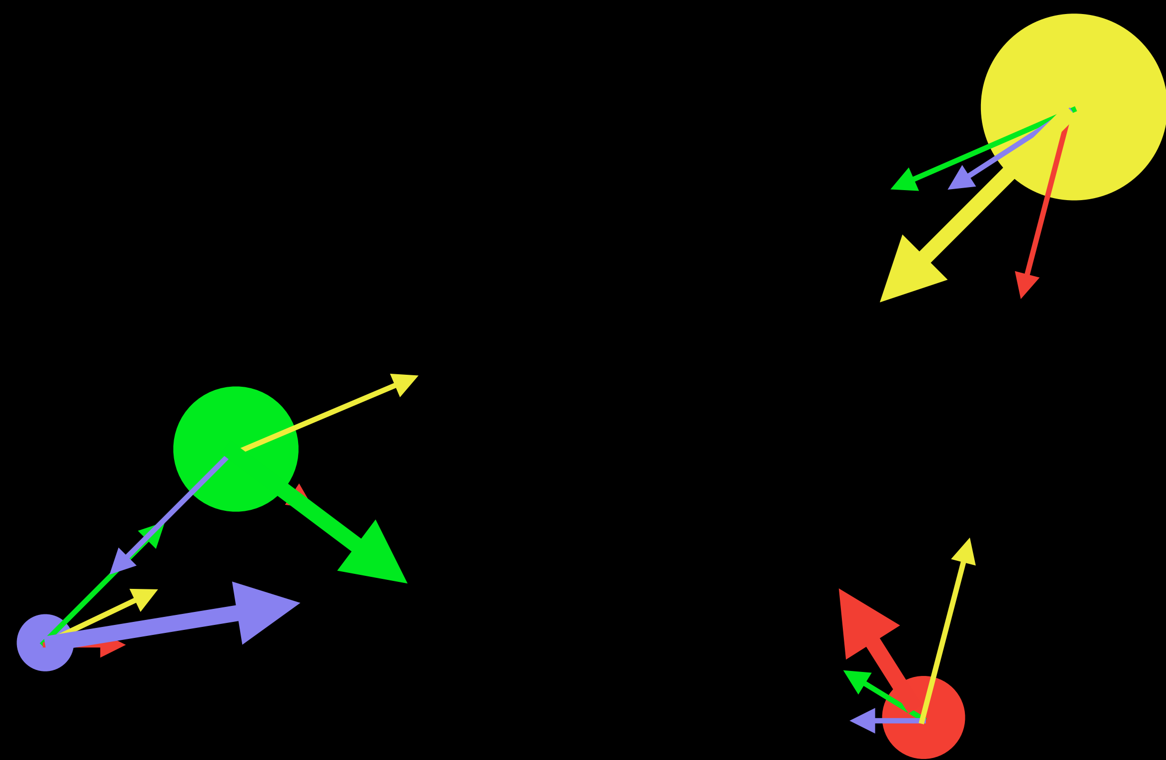
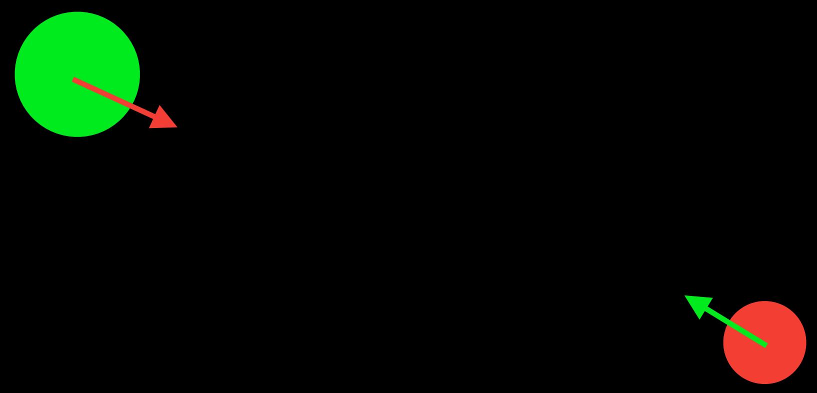


$$\mathbf{F}_i = \sum_{\substack{1 \le j \le N \\ j \ne i}} \mathbf{f}_{ij} = G m_i \cdot \sum_{\substack{1 \le j \le N \\ j \ne i}} \frac{m_j \mathbf{r}_{ij}}{\left\| \mathbf{r}_{ij} \right\|^3}.$$

Force between any set of particles ~ $(M \times m)/r^2$

N-Body Simulations: The 2 Body problem



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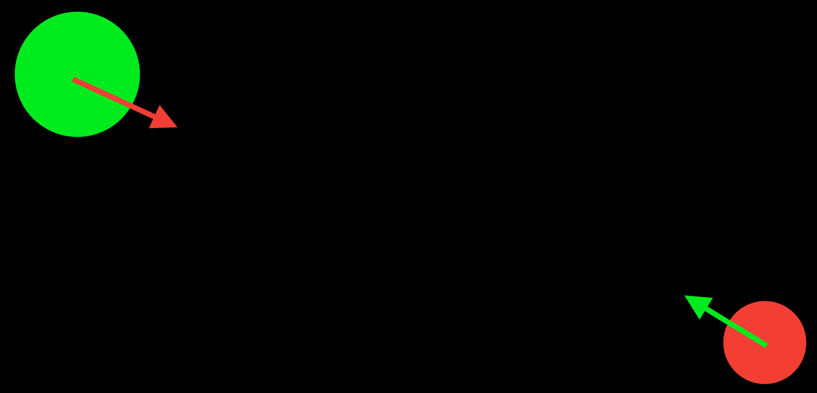

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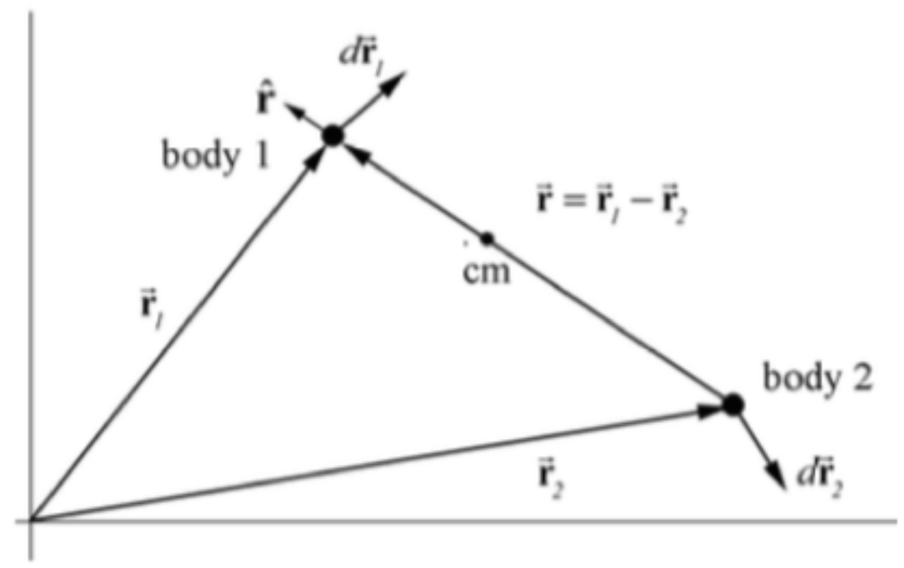
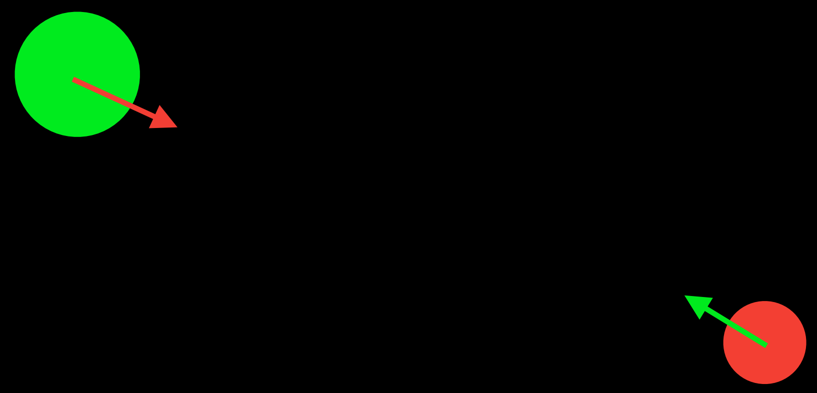


Figure 17.2 Coordinate system for the two-body problem.



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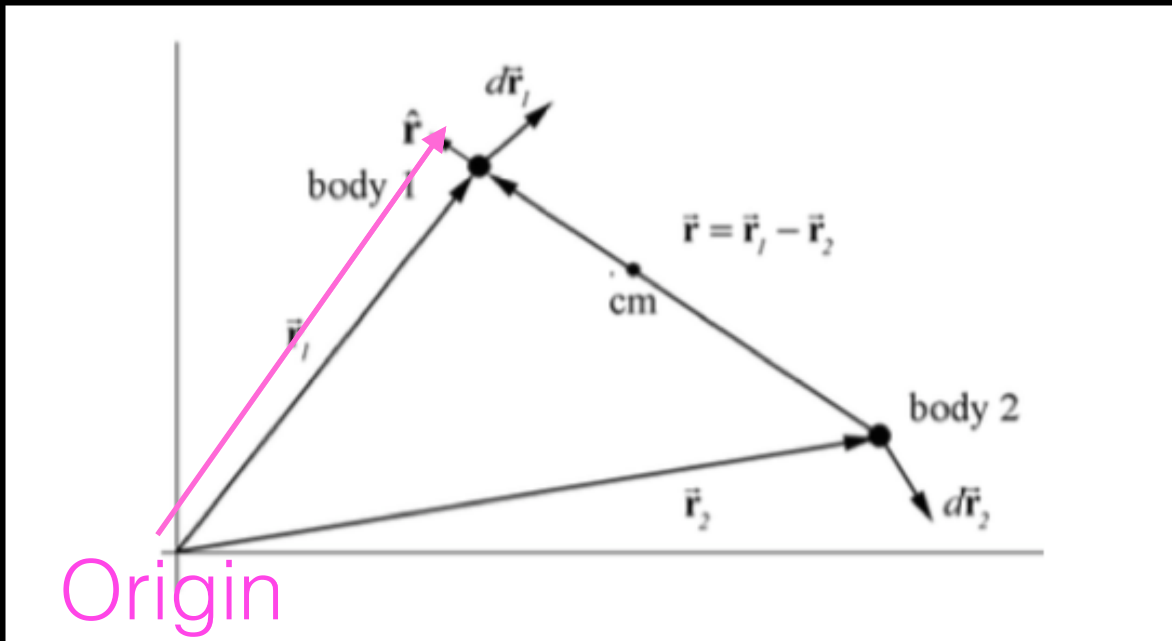

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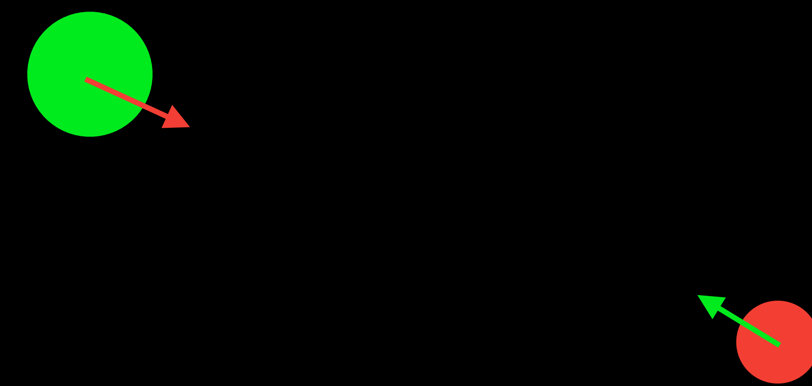

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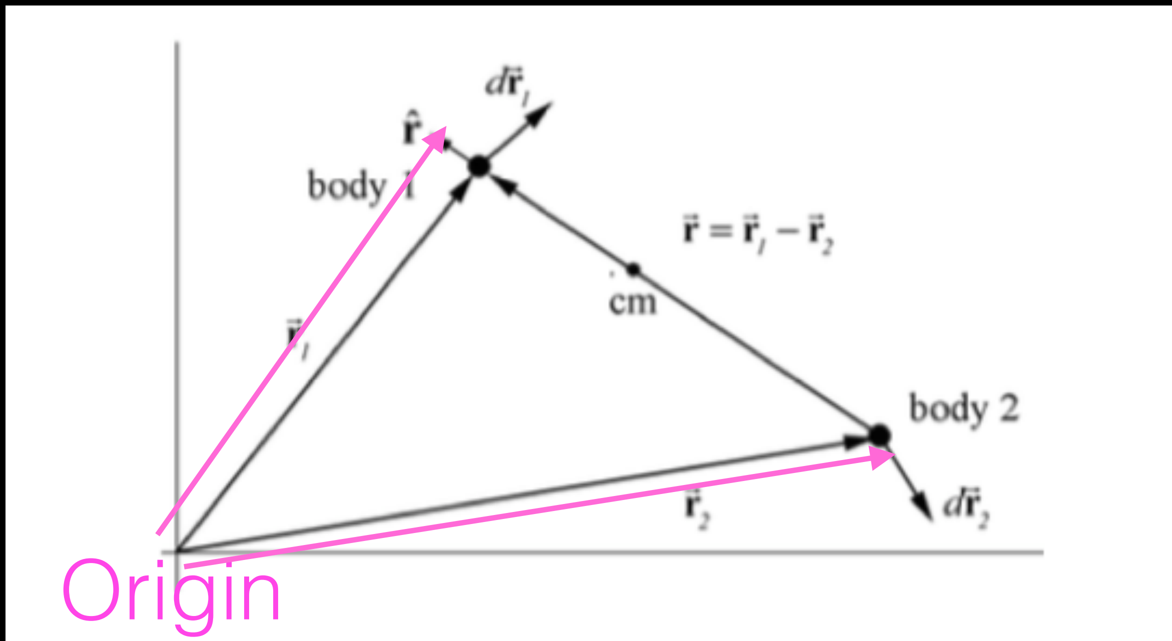

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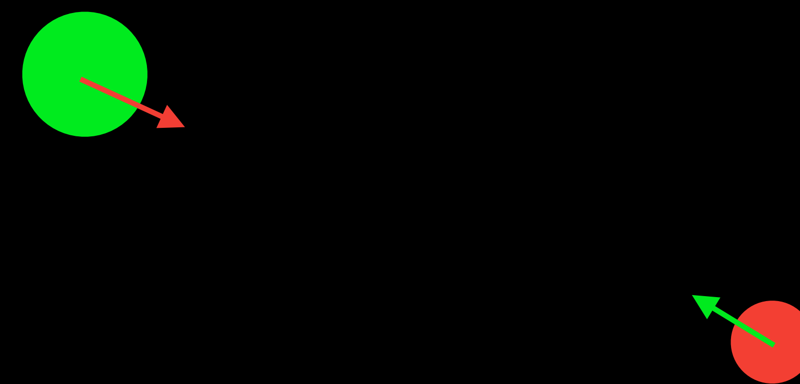

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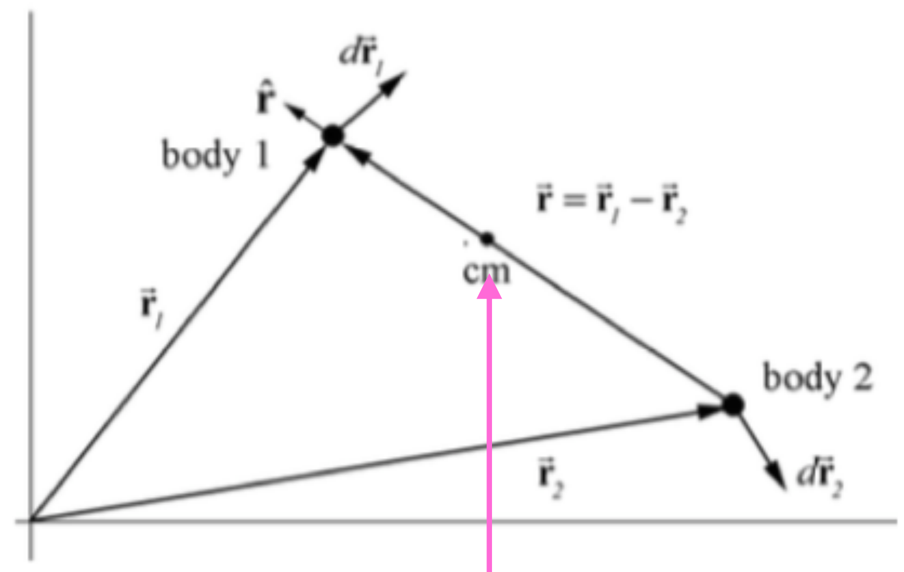

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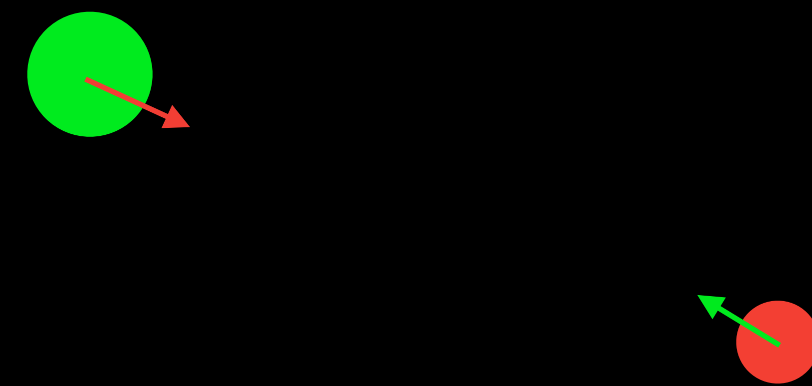
center of mass = balance point



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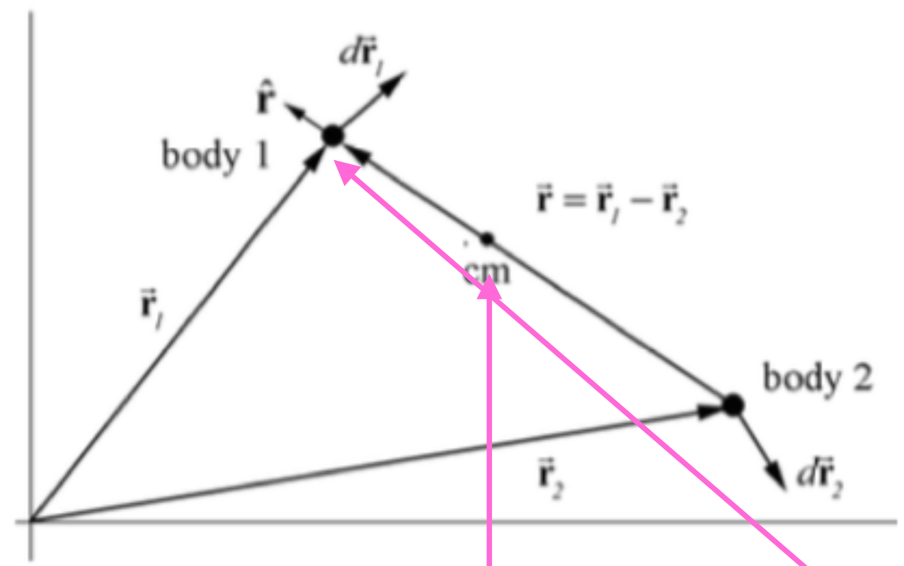

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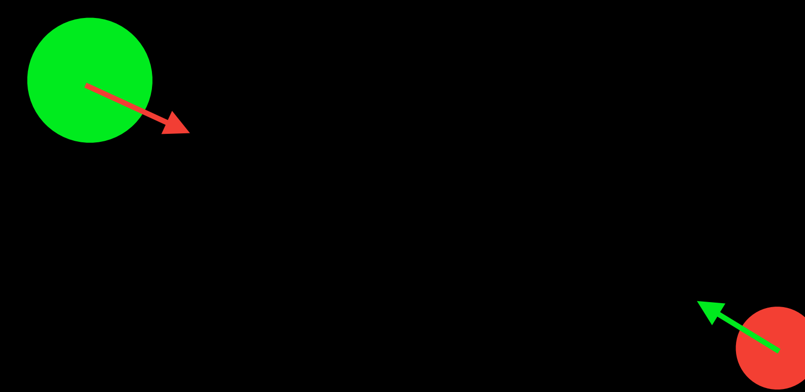

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body 1 experiences an acceleration from the gravitational force between it and body 2

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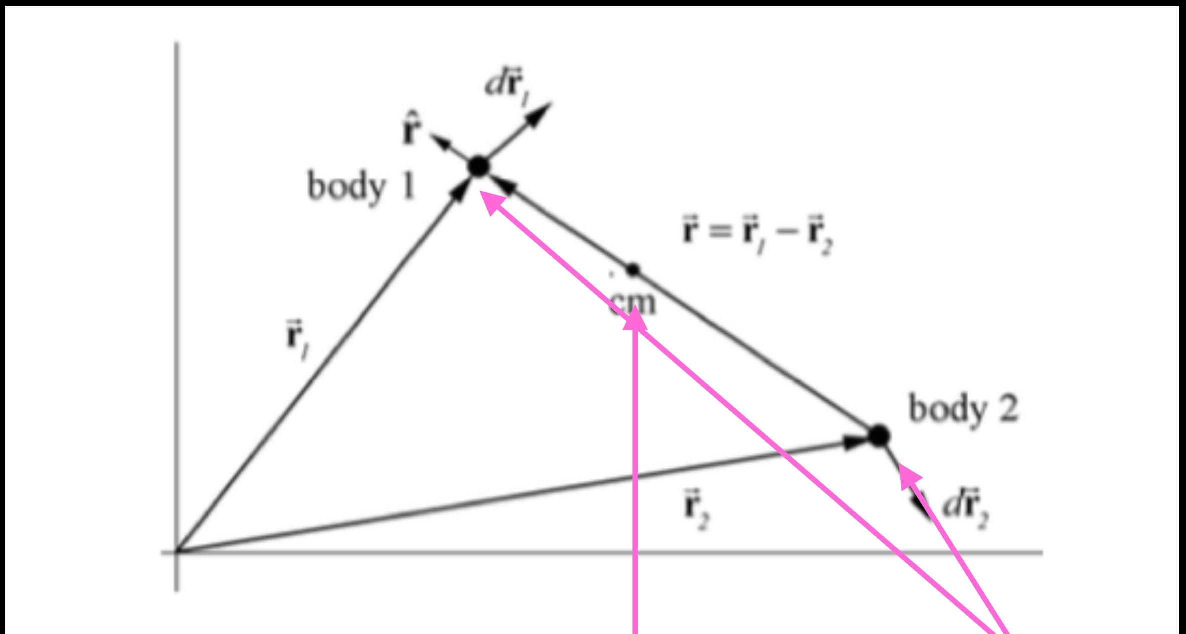

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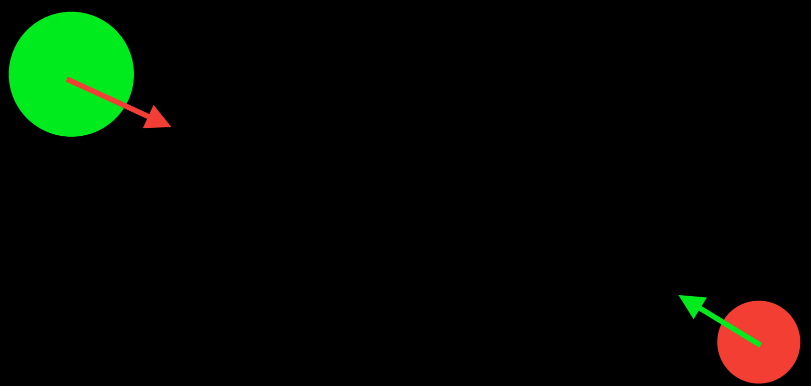

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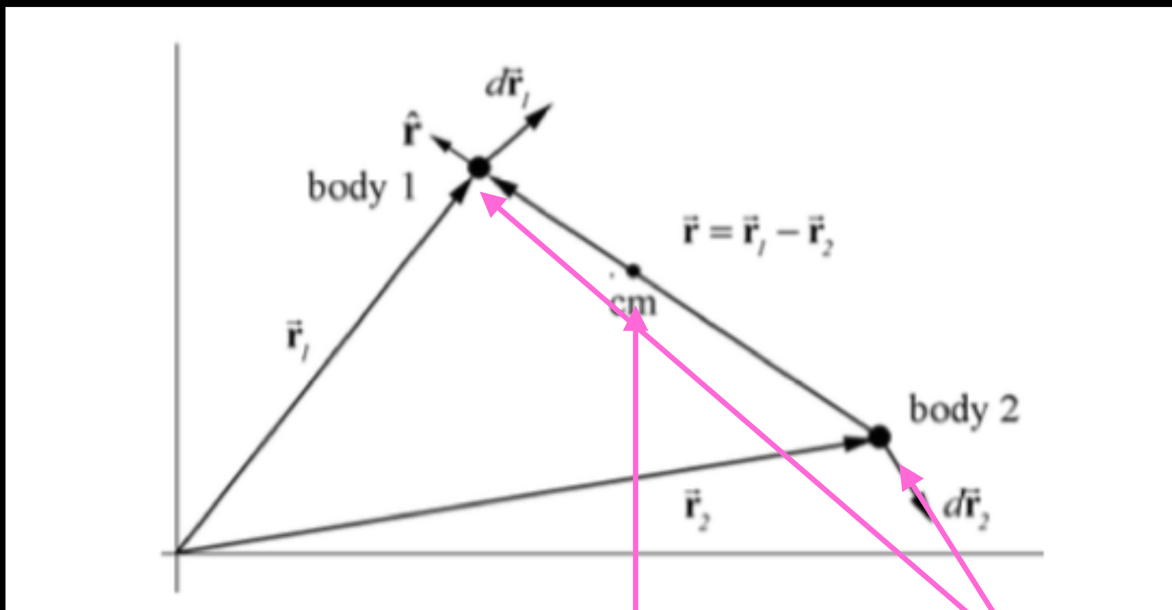

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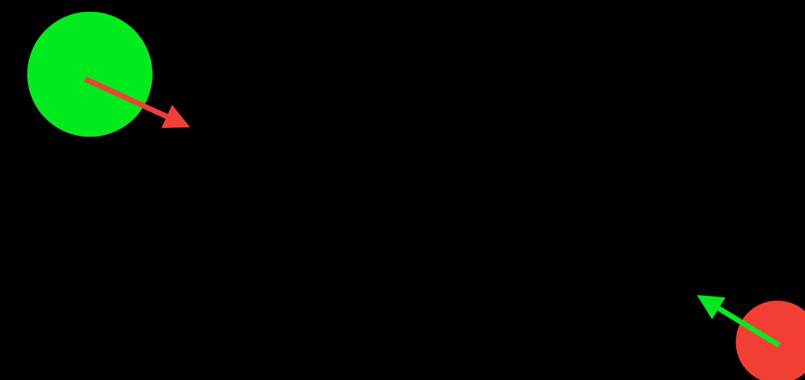

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center of mass = balance point

Where gravity depends on their masses and the distance between them.

body 1 experiences an acceleration from the gravitational force between it and body 2

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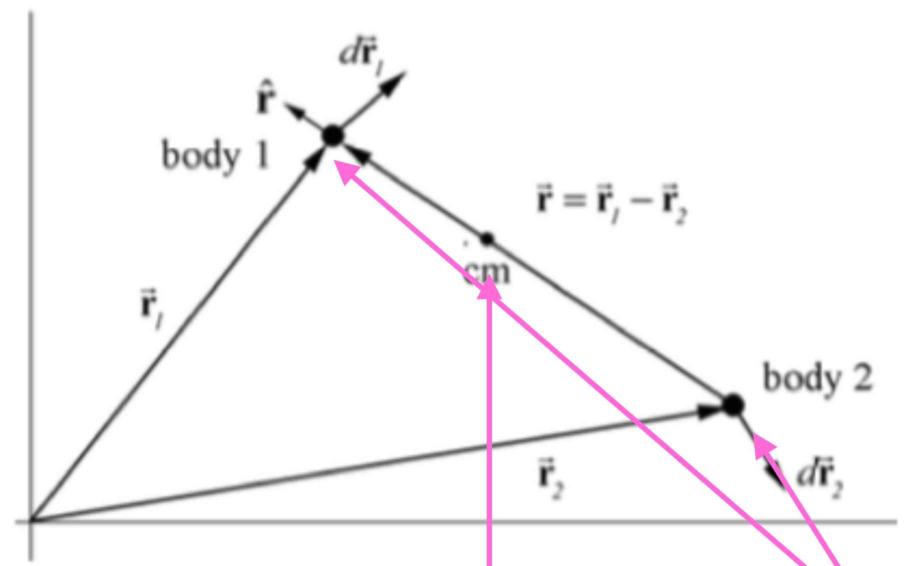
N-Body Simulations: The 2 Body problem



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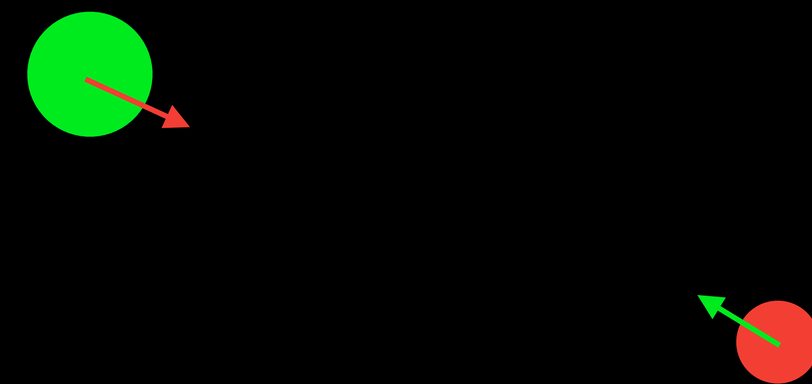

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center of mass = balance point

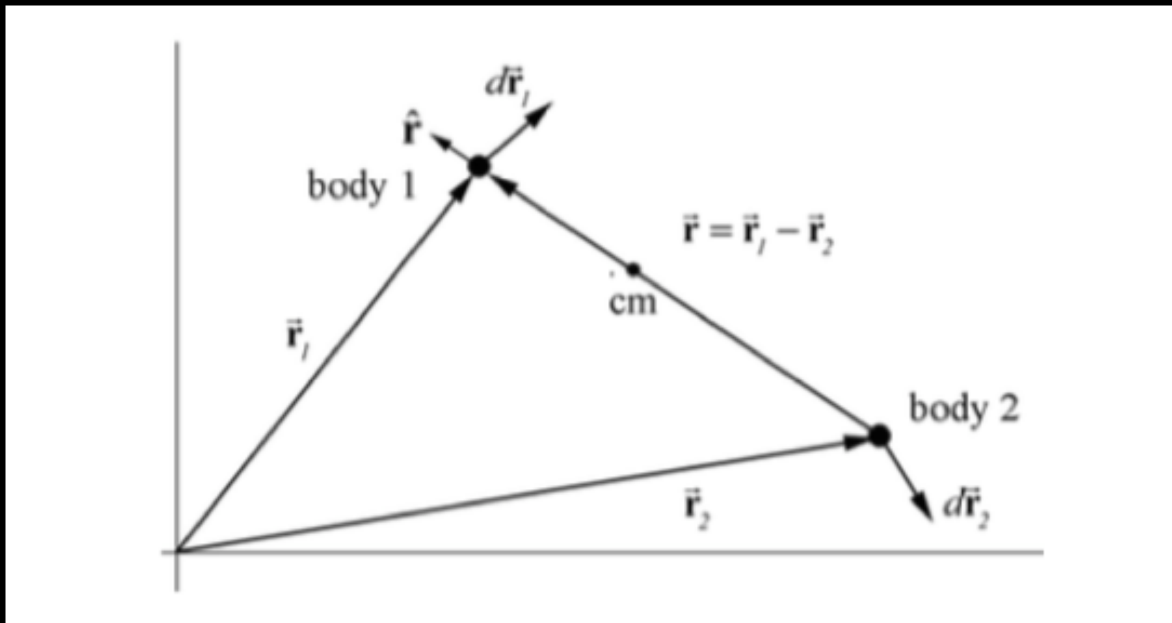
Where gravity depends on their masses and the distance between them.

body 1 experiences an acceleration from the gravitational force between it and body 2

The 2nd derivative of a position depends on the position through the force of gravity - **Differential Equation!**

body 2 experiences an acceleration from the gravitational force between it and body 1

N-Body Simulations: The 2 Body problem



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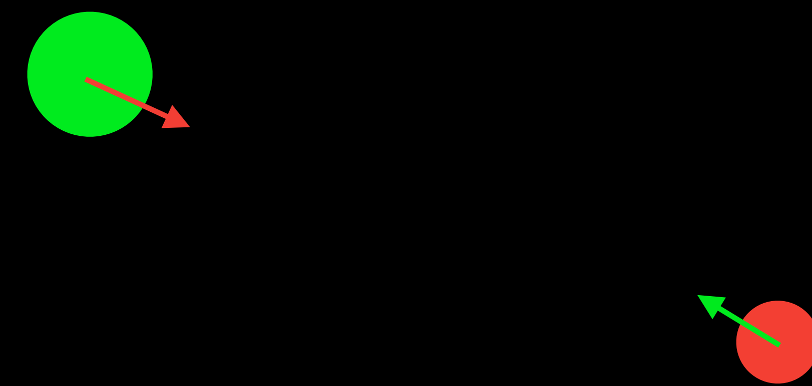

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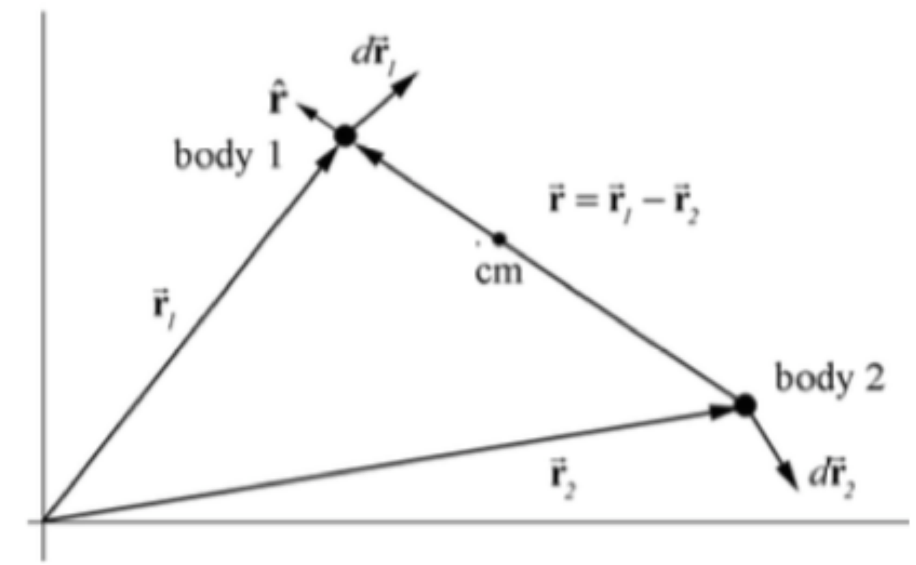

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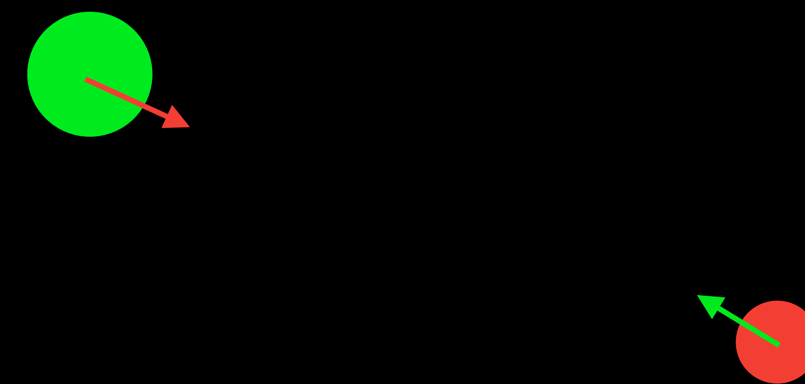
A BUNCH OF MATH
(ask Jenny!)



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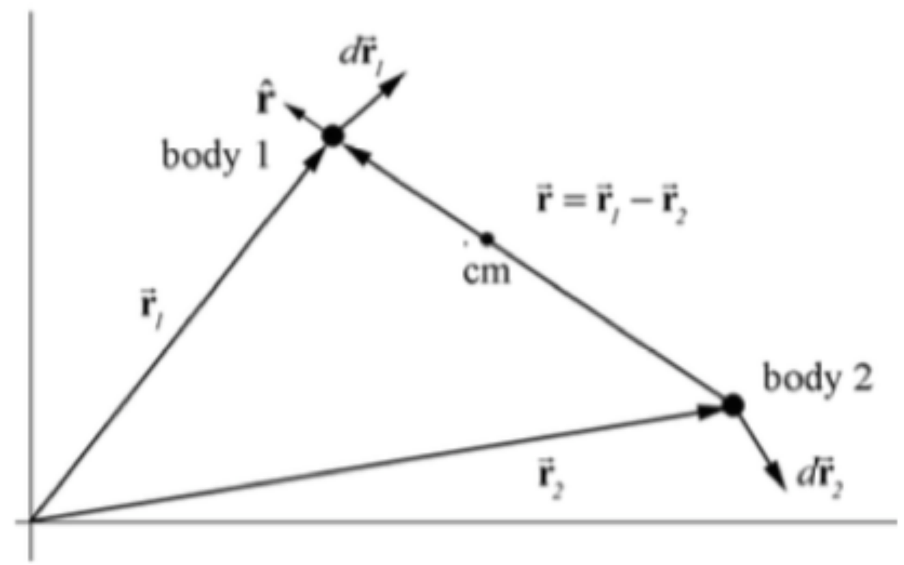

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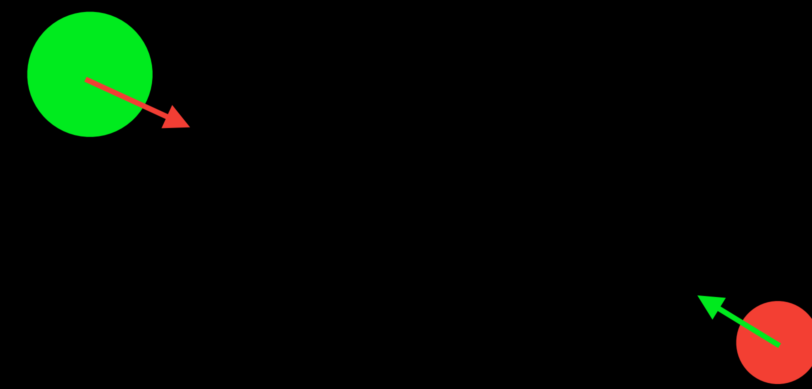

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A BUNCH OF MATH
(ask Jenny!)

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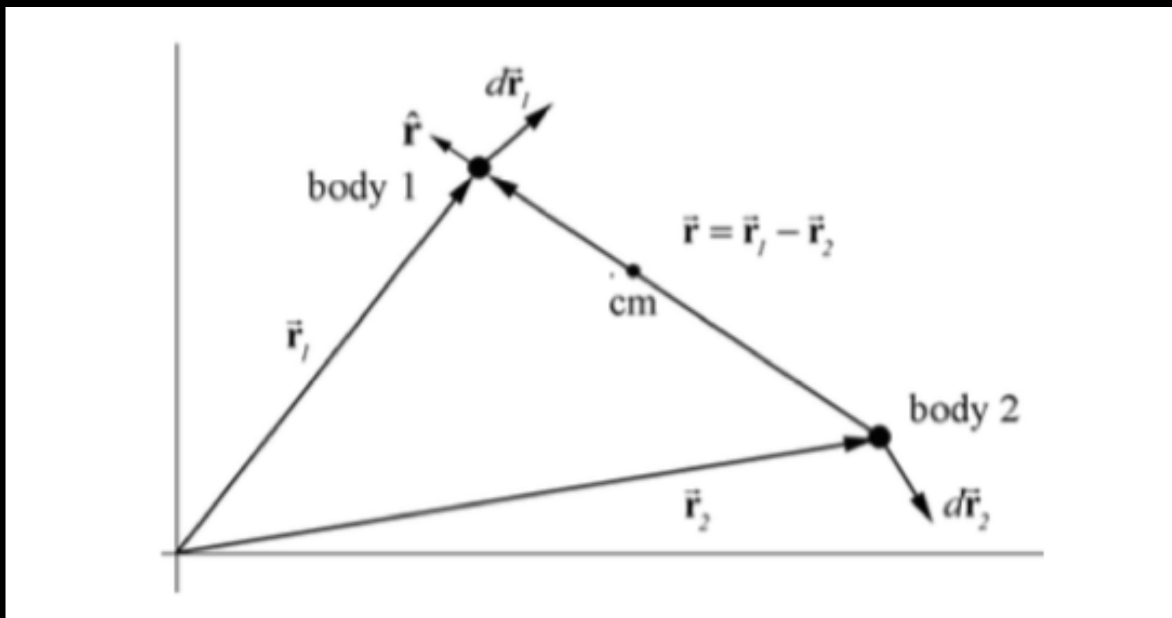

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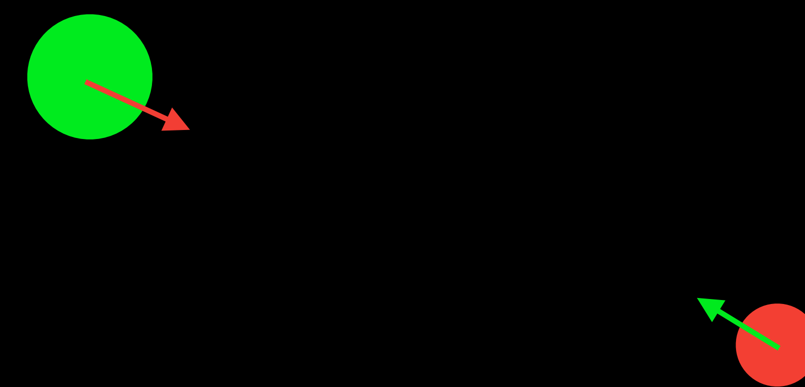

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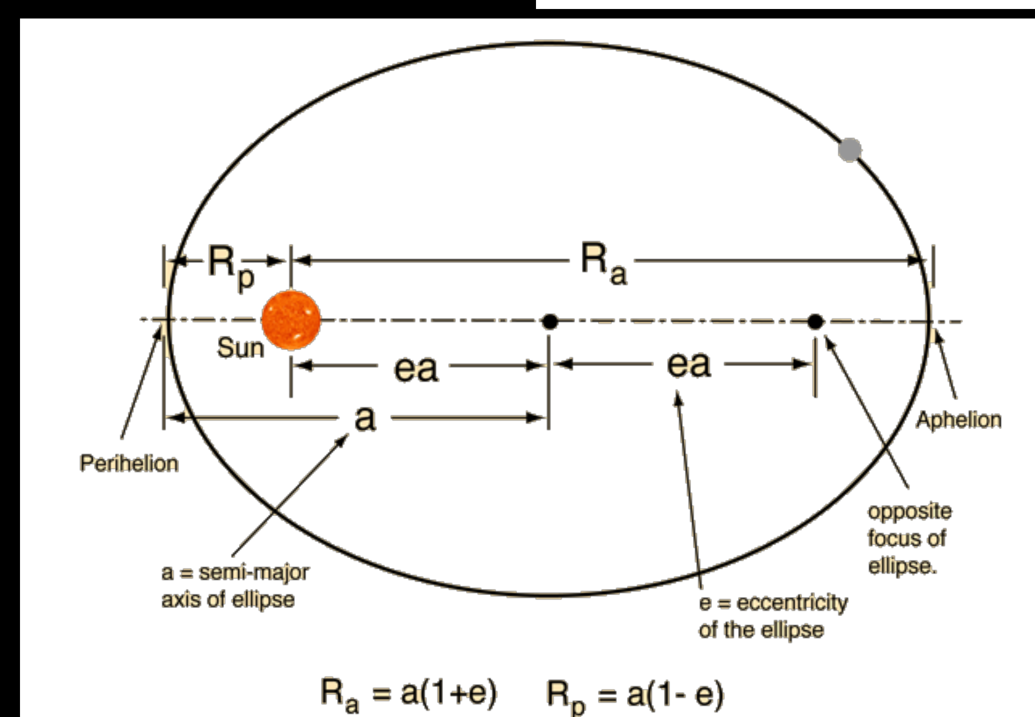


N-Body Simulations: The 2 Body problem

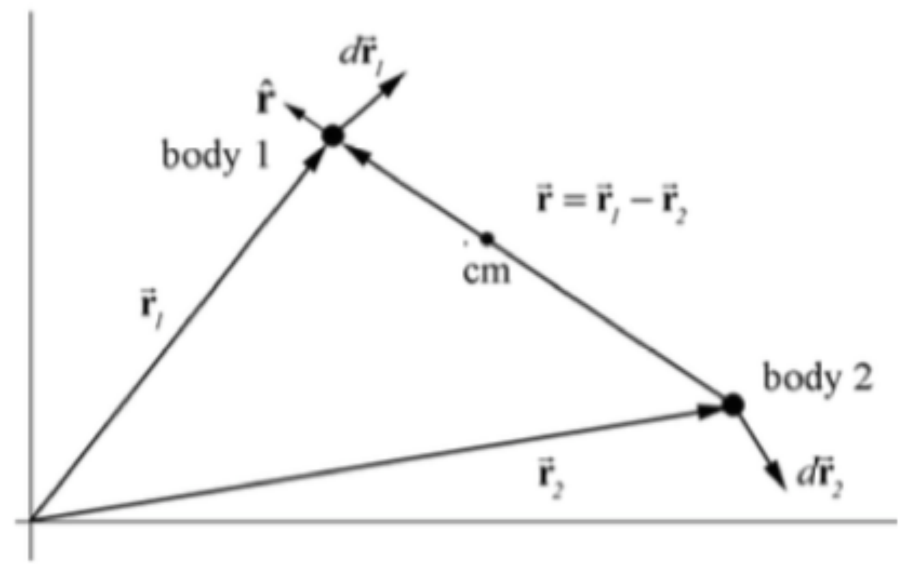
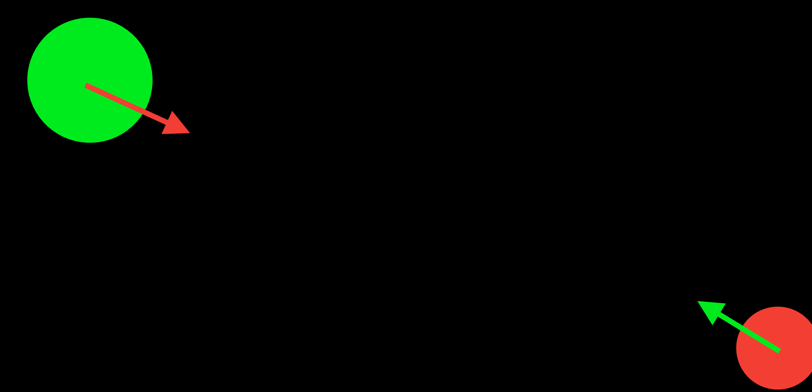


Figure 17.2 Coordinate system for the two-body problem.



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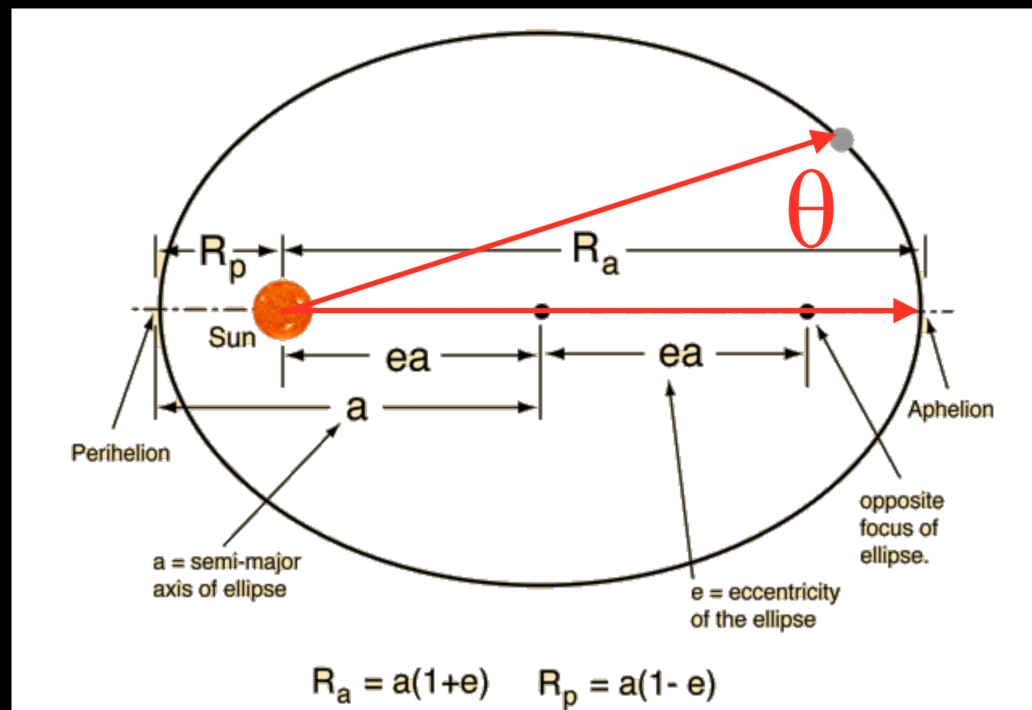
A BUNCH OF MATH
(ask Jenny!)

$$r = \frac{a(1-e^2)}{1 + e\cos\theta}$$



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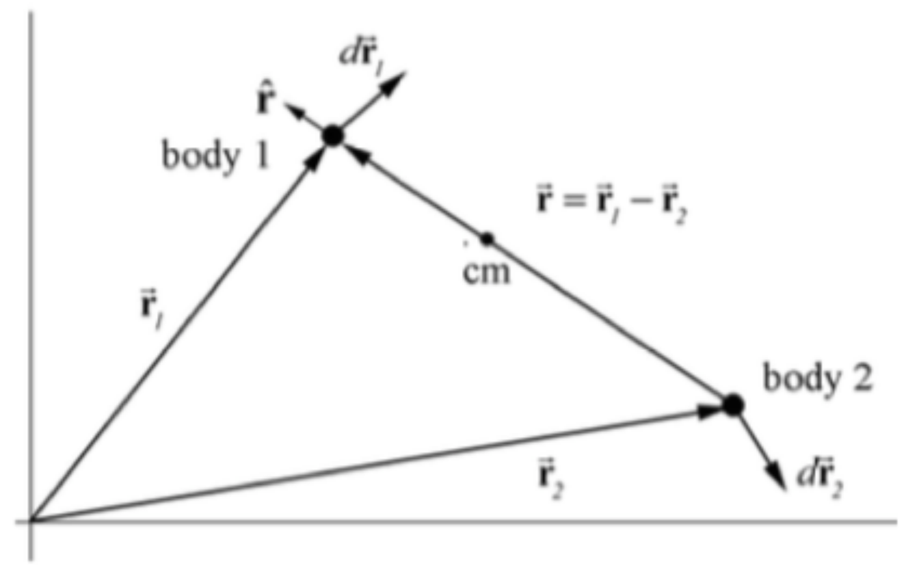

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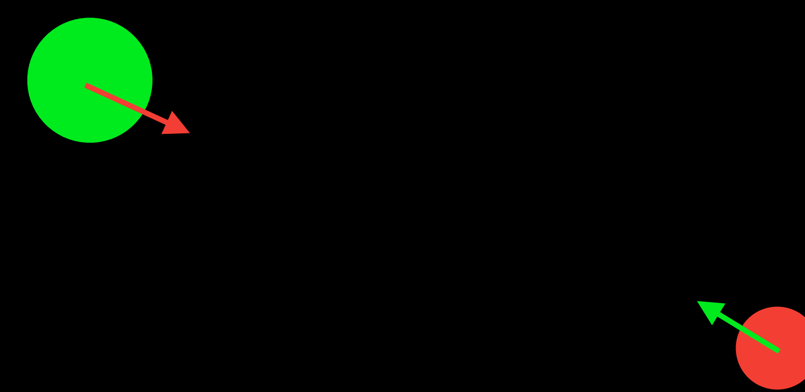

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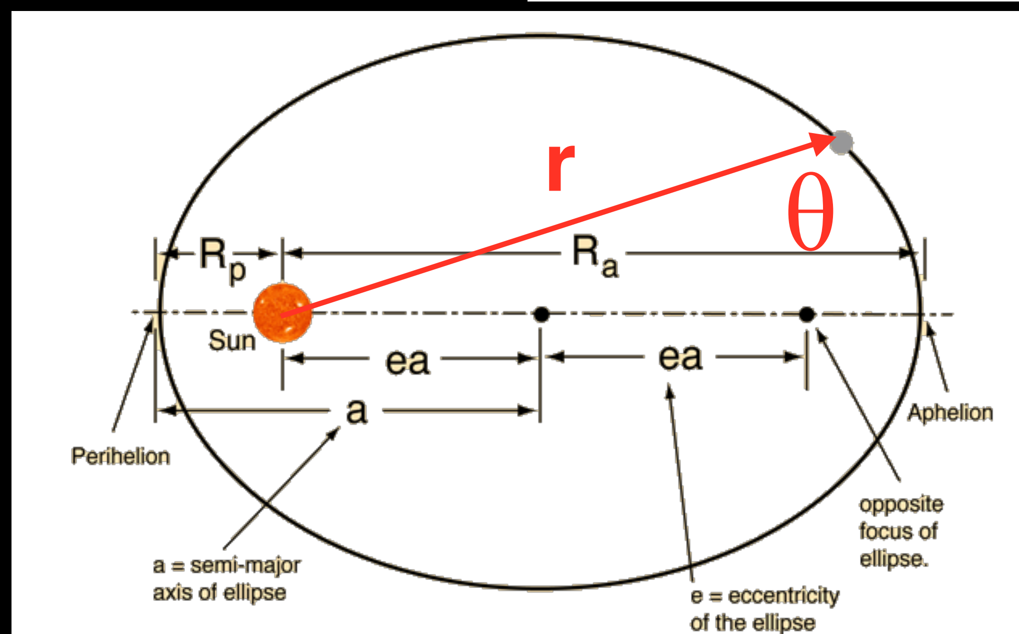
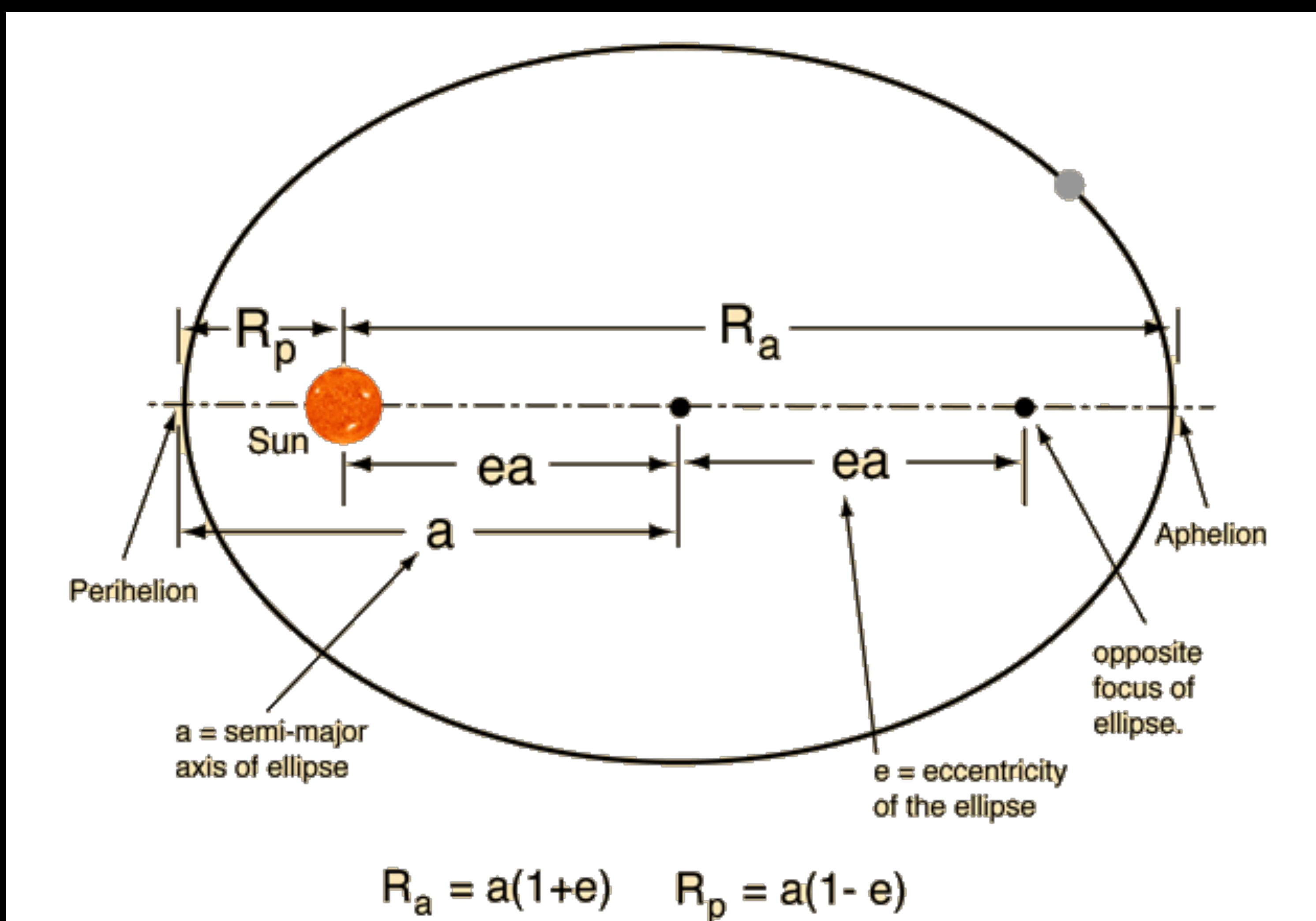


$R_a = a(1+e)$ $R_p = a(1-e)$

N-Body Simulations: The 2 Body problem

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$



Bunch of relations between: e, a, Ra, Rp, etc (more info in links online)

N-Body Simulations: The 2 Body problem

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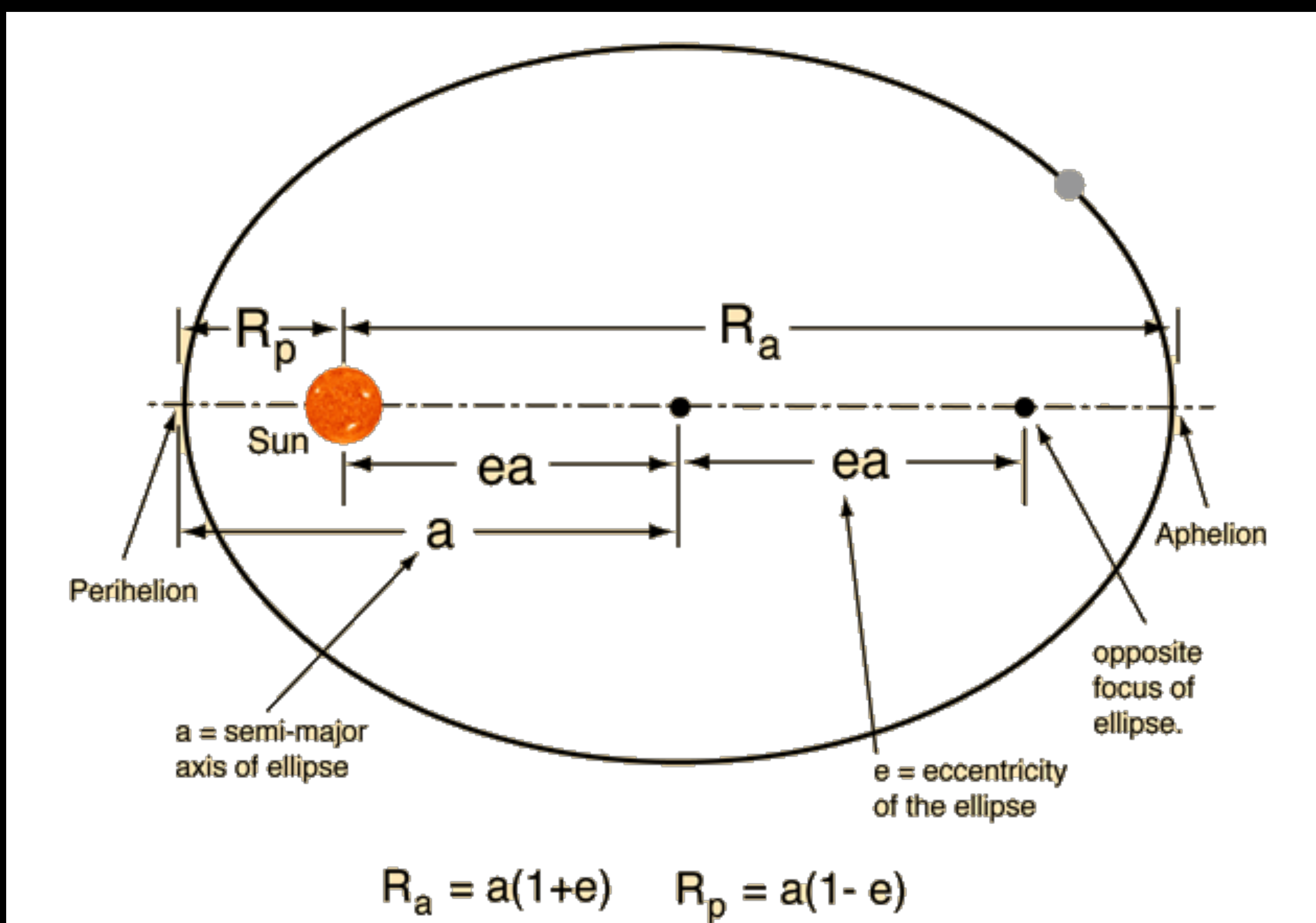


One super important one is Kepler's 3rd Law

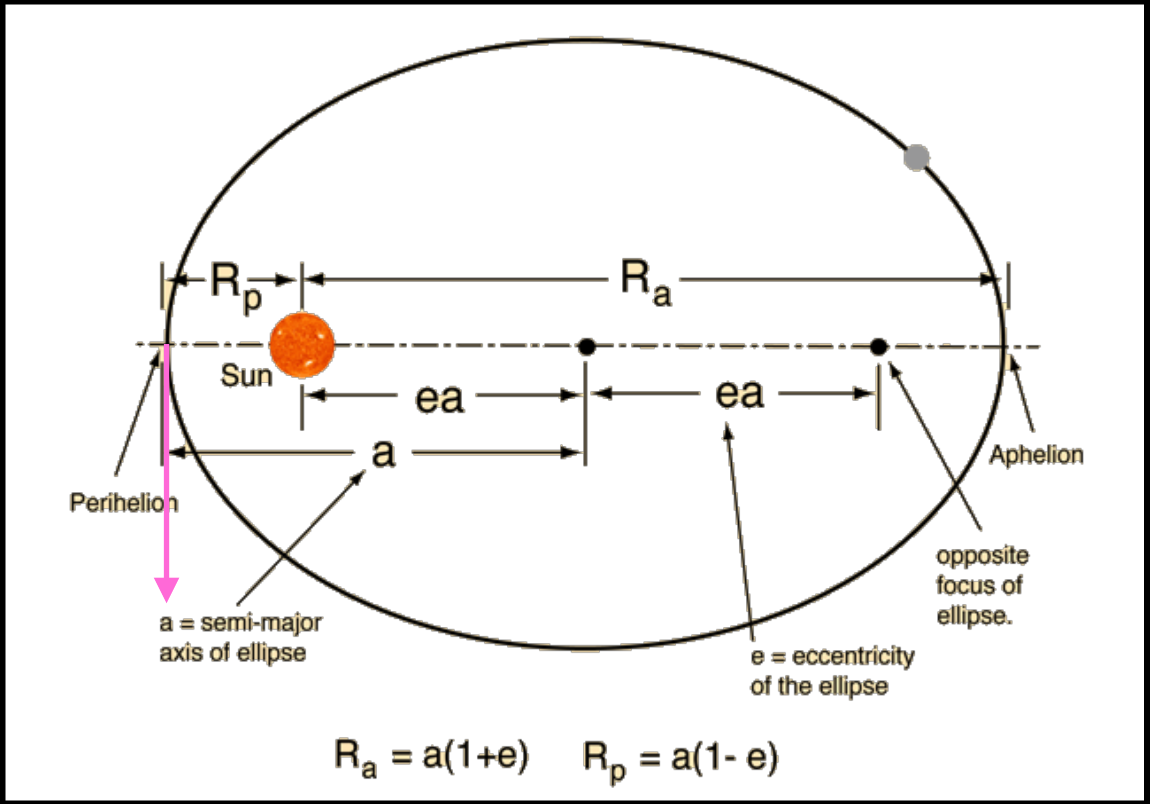
$$\frac{P^2}{a^3} = \frac{4\pi^2}{G(M+m)}$$

Period = time to go around once

Bunch of relations between: e, a, Ra, Rp, etc (more info in links online)

N-Body Simulations: The 2 Body problem

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$



$$\frac{P^2}{a^3} = \frac{4\pi^2}{G(M+m)}$$

Some conserved quantities:

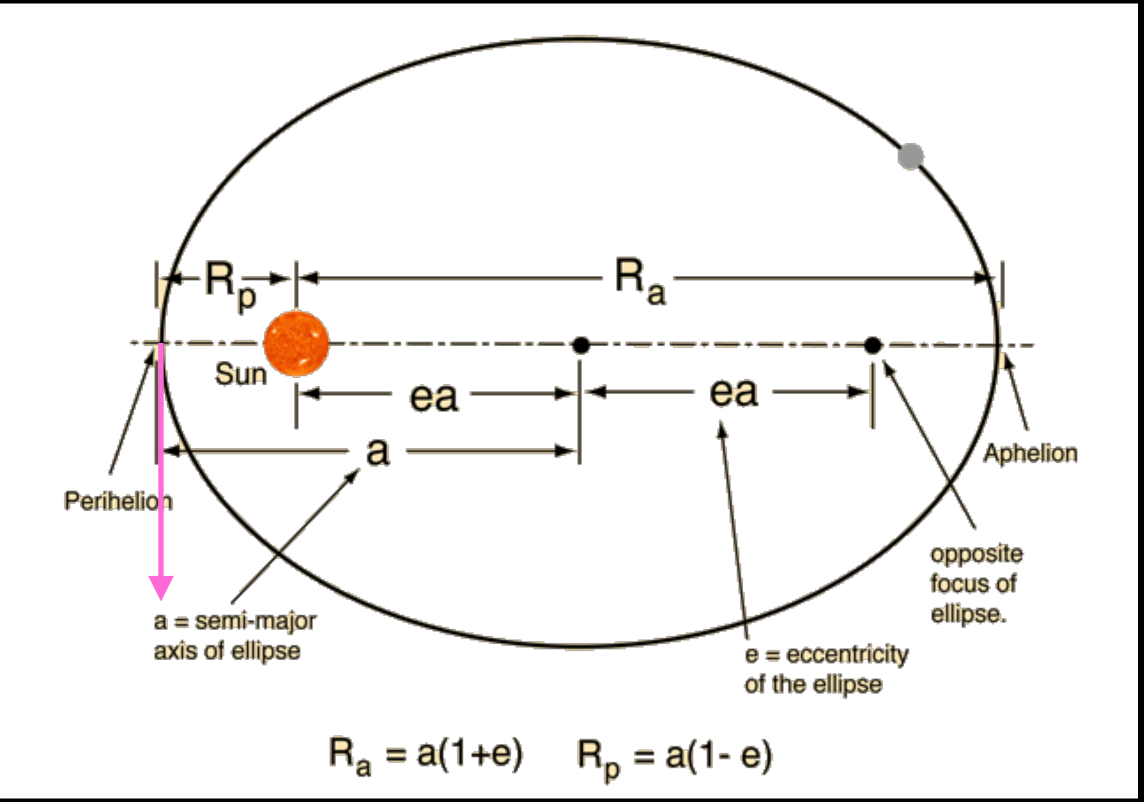
$$E = -\frac{Gm_1 m_2}{2a}$$

$$\vec{L} = m_1 \vec{R}_{P,1} \times \vec{v}_{P,1} + m_2 \vec{R}_{P,2} \times \vec{v}_{P,2}$$

Derivations linked online! See Jenny too!

N-Body Simulations: The 2 Body problem

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

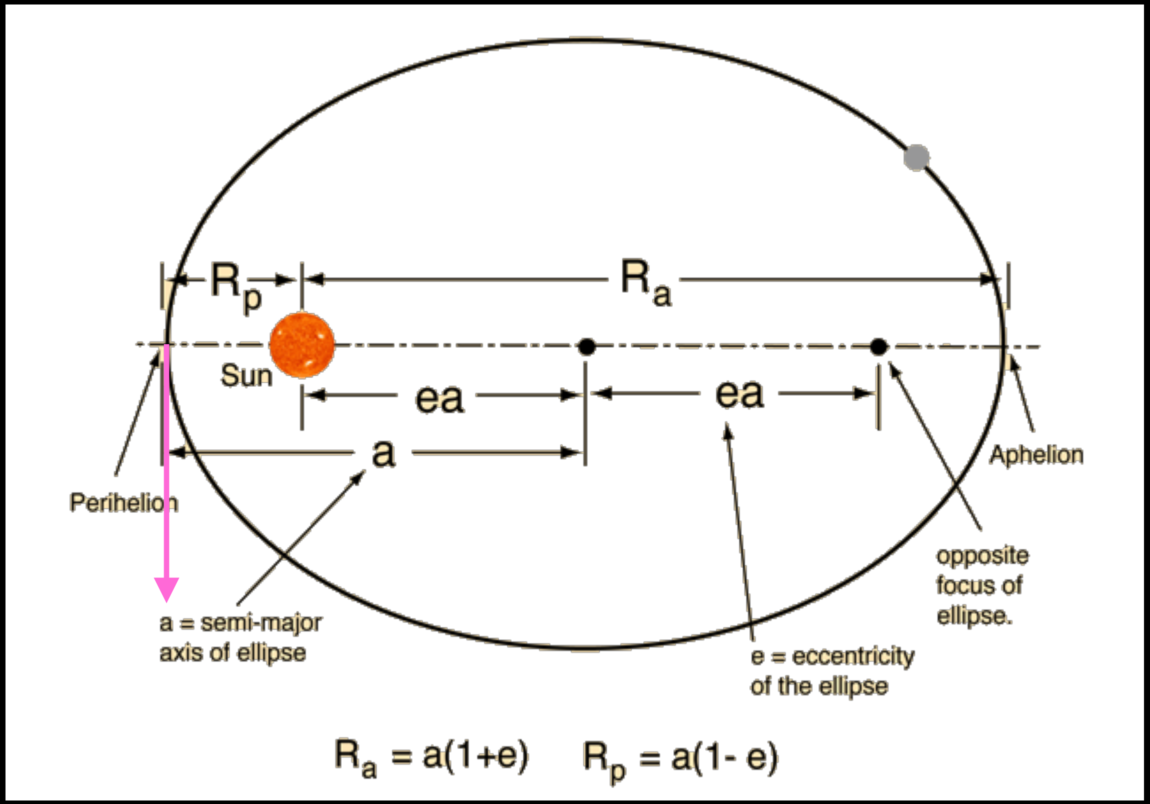


One more thing about e:

circular orbit for e = 0
elliptical orbit for 0 < e < 1 **bound**
parabolic trajectory for e = 1 **“bound”**
hyperbolic trajectory for e > 1 **unbound**

N-Body Simulations: The 2 Body problem

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$



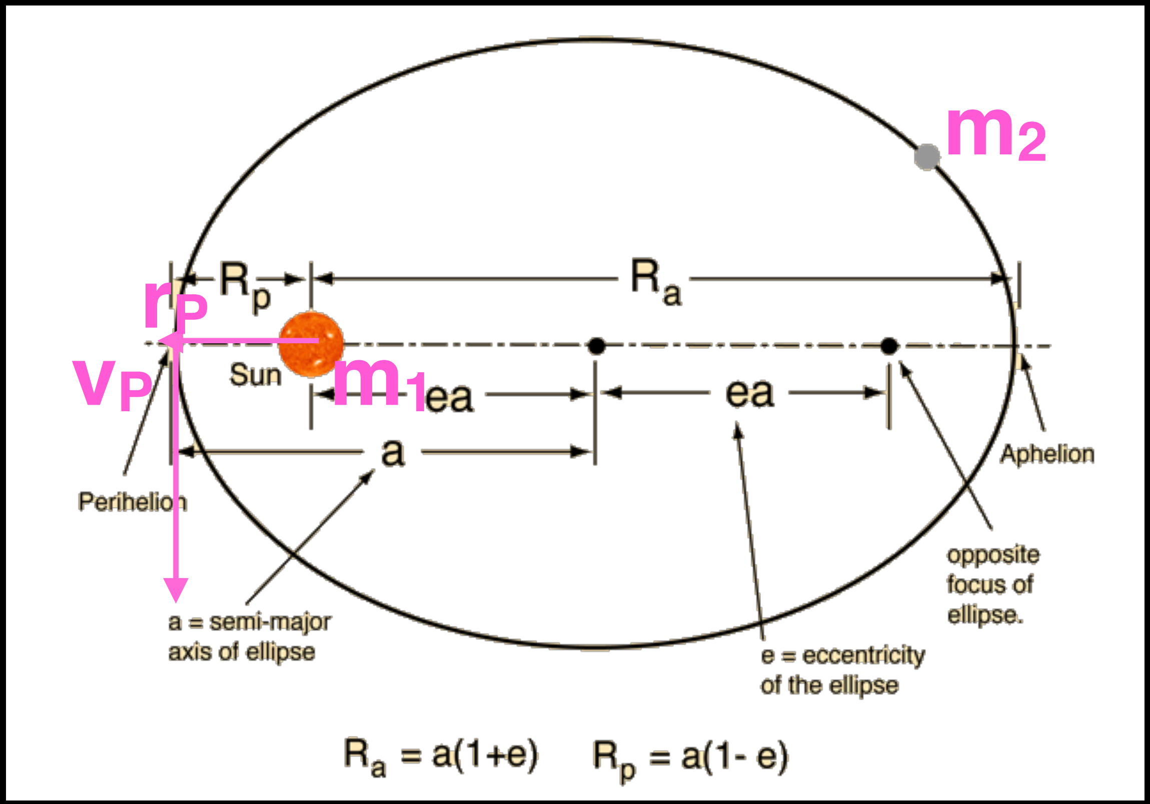
One more thing about e:

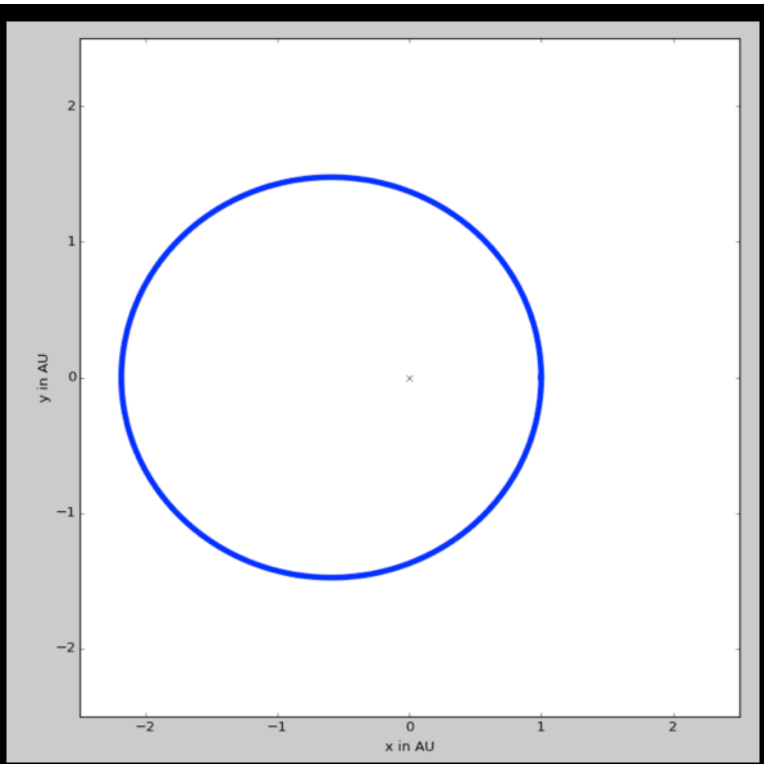
circular orbit for $e = 0$	**bound**
elliptical orbit for $0 < e < 1$	
parabolic trajectory for $e = 1$	**“bound”**
hyperbolic trajectory for $e > 1$	**unbound**

N-Body Simulations: Start with 2 Bodies

```
import numpy as np
import matplotlib.pyplot as plt


# mass of particle 1 in solar masses
m1 = 1.0
# mass of particle 2 in jupiter masses
m2 = 1.0
# distance of m2 at closest approach (pericenter)
rp = 1.0 # in AU
# velocity of m2 at this closest approach distance
# we assume vp of the larger mass (m1) is negligable
vp = 35.0 # in km/s

# analytically here are the constants we need to define to solve:
ecc = rp*vp*vp/(G*(m1+m2)) - 1.0
a = rp/(1.0 - ecc)

# now, generate the theta array
ntheta = 500 # number of points for theta
th_an = np.linspace(0, 360, ntheta)

# now, create r(theta)
r_an = (a*(1-ecc*ecc))/(1.0 + ecc*np.cos( th_an*np.pi/180.0 ))

# for plotting -> x/y coords for m2
x_an = r_an*np.cos( th_an*np.pi/180.0 )/AUinCM
y_an = r_an*np.sin( th_an*np.pi/180.0 )/AUinCM

# plot x/y coords
fig, ax = plt.subplots(1, figsize = (10, 10))

ax.plot(x_an, y_an, linewidth=5)

plt.show()
```

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$





Code link at: www.astroblend.com/ba2016/day1.html

N-Body Simulations

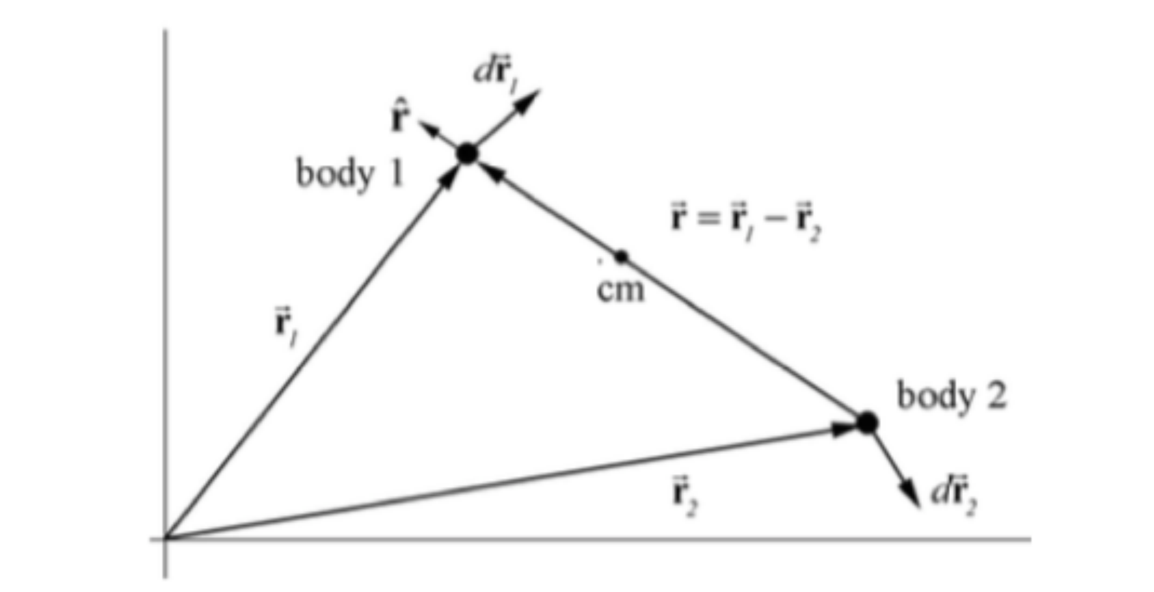
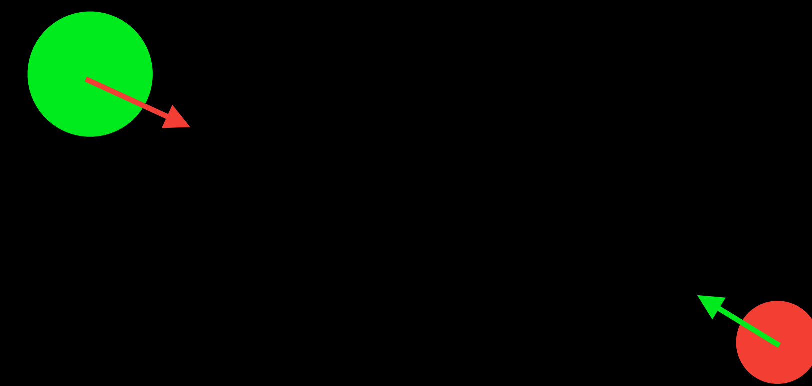


Figure 17.2 Coordinate system for the two-body problem.



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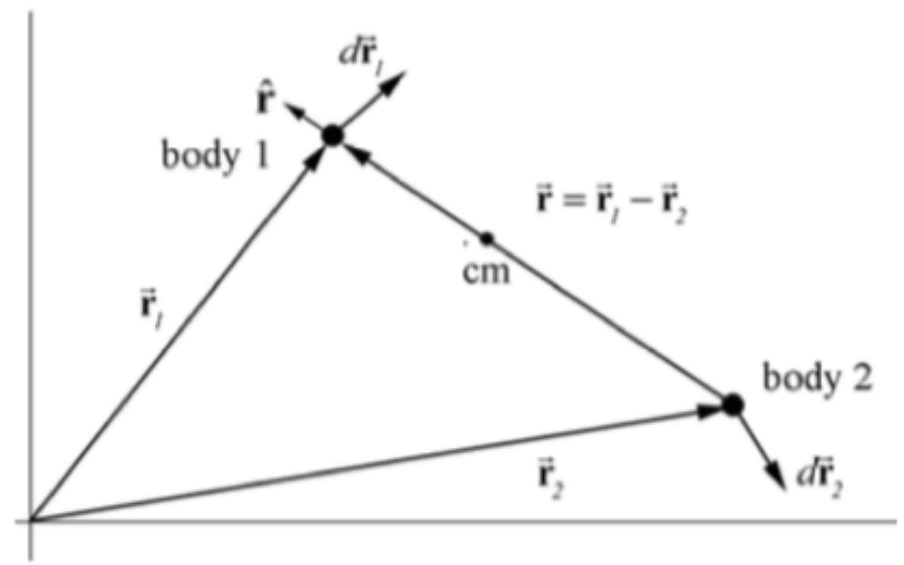
N-Body Simulations



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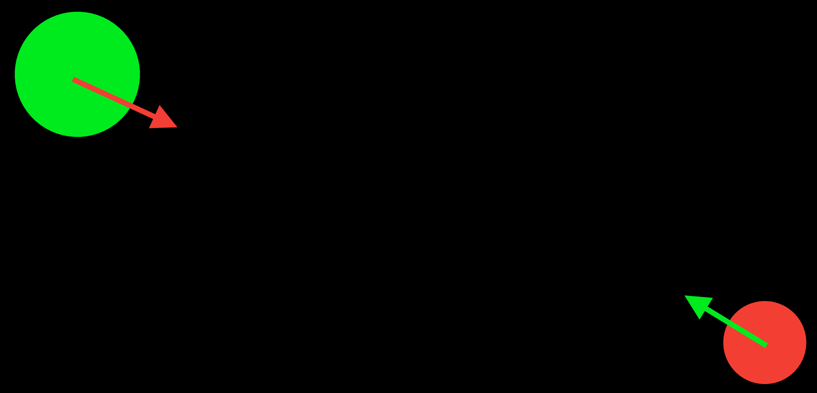

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Ok, but what if we want to solve this problem numerically…

N-Body Simulations

Lets say we look over a small Δt - such that over this small amount of time both the velocity are approximately constant. We know from calculus…

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int_{v_0}^{v} dv = \int_{0}^{\Delta t} a\,dt$$

$$v - v_0 = a\Delta t$$

$$v = v_0 + a\Delta t$$

$$v = \frac{dx}{dt}$$

$$dx = v\,dt = (v_0 + at)\,dt$$

$$\int_{x_0}^{x} dx = \int_{0}^{\Delta t} (v_0 + at)\,dt$$

$$x - x_0 = v_0\Delta t + ½a\Delta t^2$$

$$x = x_0 + v_0\Delta t + ½a\Delta t^2$$

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$$\int_{x_0}^{x} dx = \int_{0}^{\Delta t} (v_0 + at)\,dt$$

$$x - x_0 = v_0\Delta t + \tfrac{1}{2}a\Delta t^2$$

$$x = x_0 + v_0\Delta t + \tfrac{1}{2}a\Delta t^2$$

N-Body Simulations

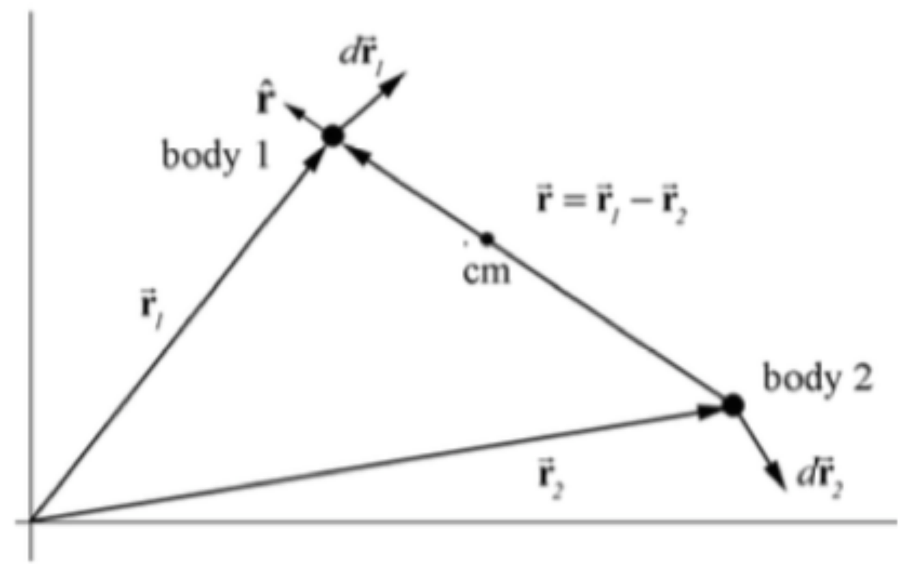
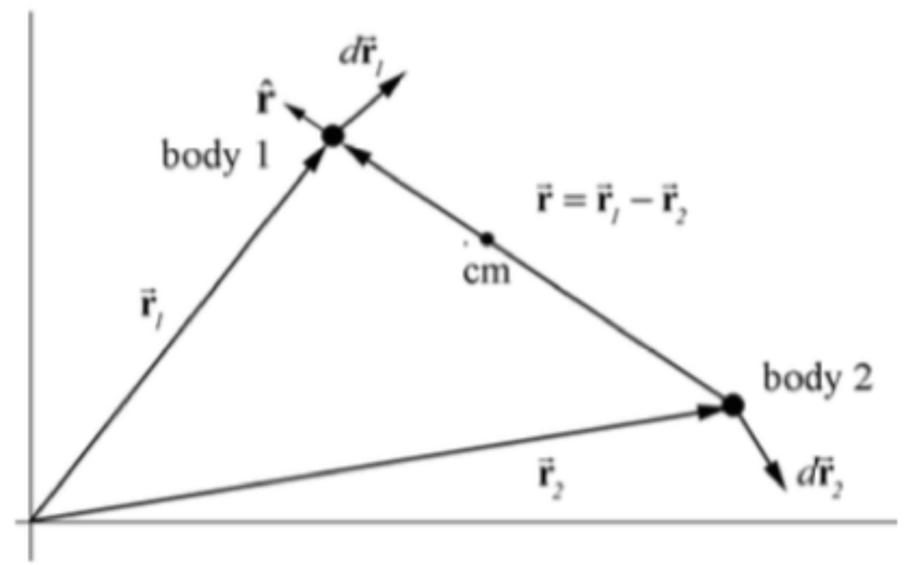


Figure 17.2 Coordinate system for the two-body problem.

An itty bitty timestep $\Delta t = t_{n+1} - t_n$

N-Body Simulations



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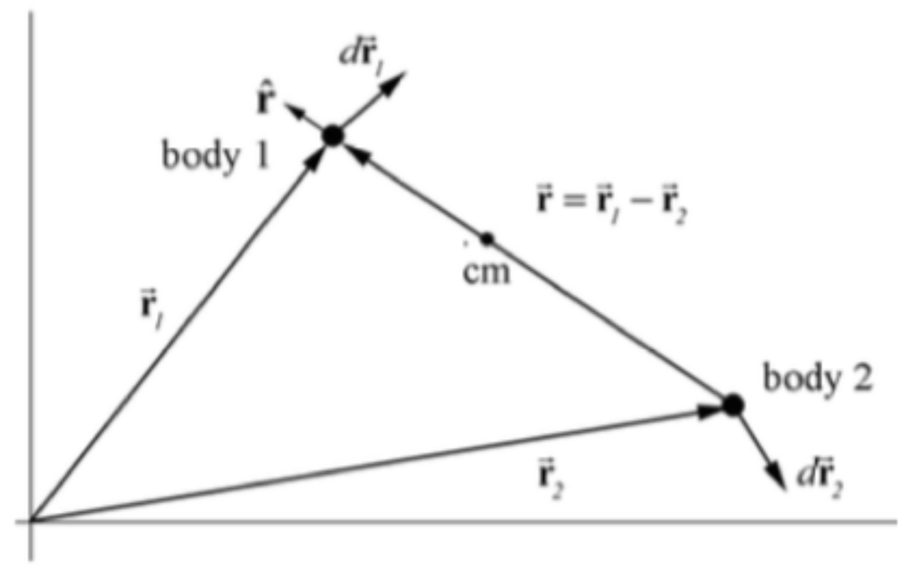
N-Body Simulations



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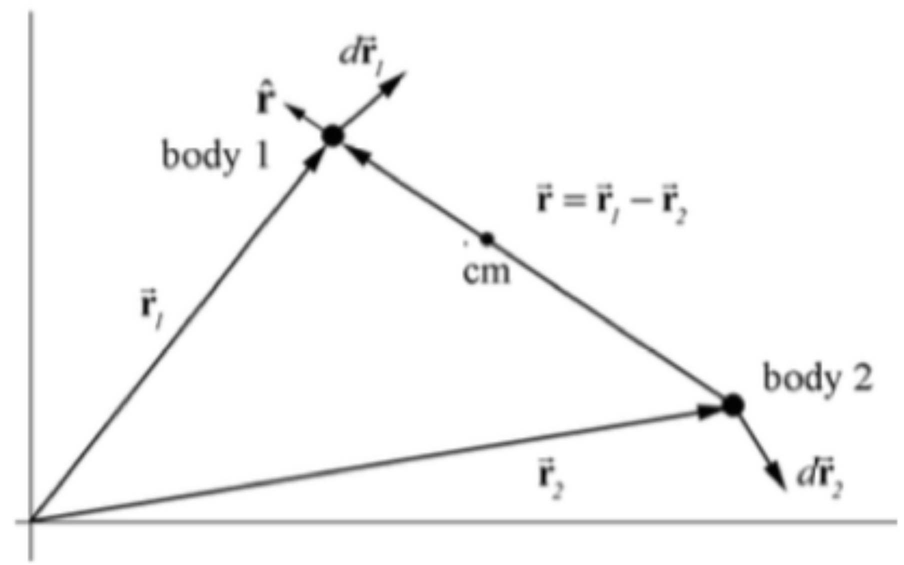
N-Body Simulations



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~ Constant

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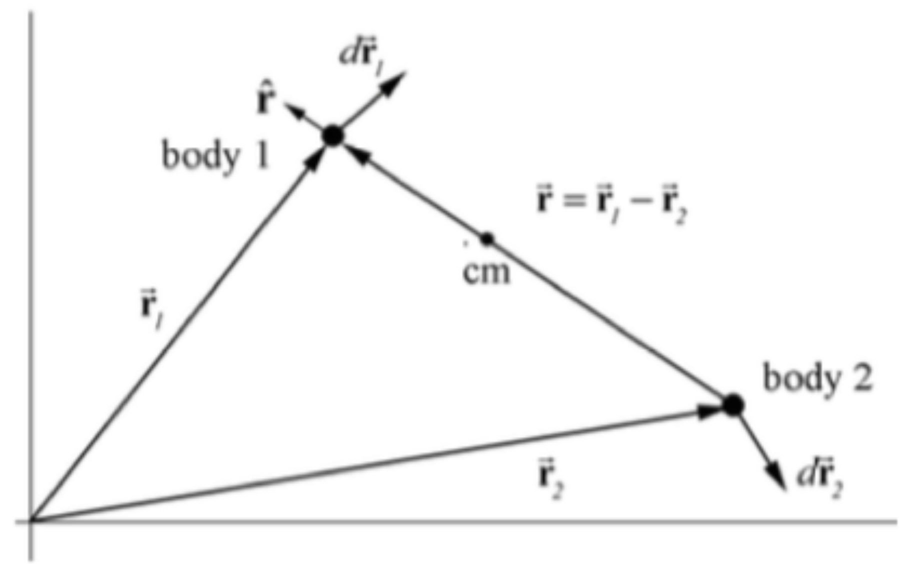

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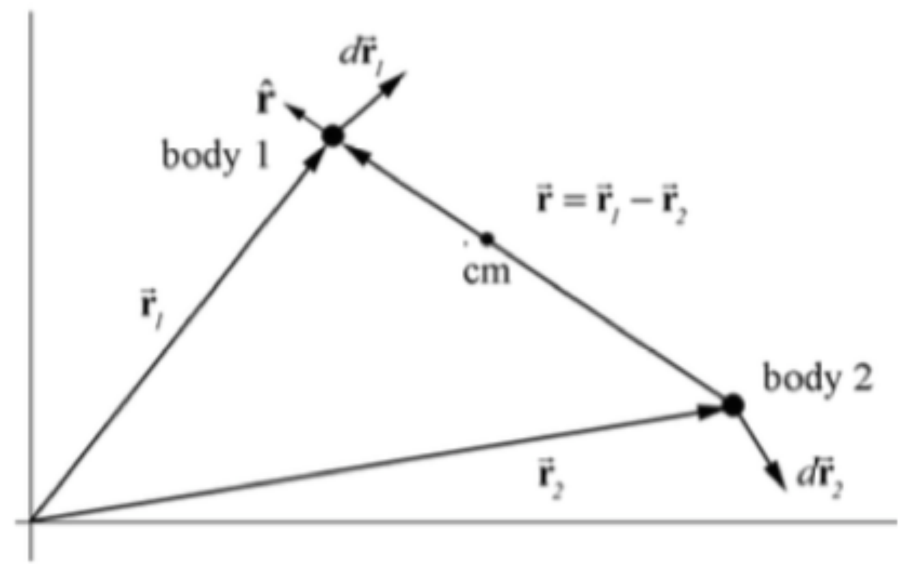

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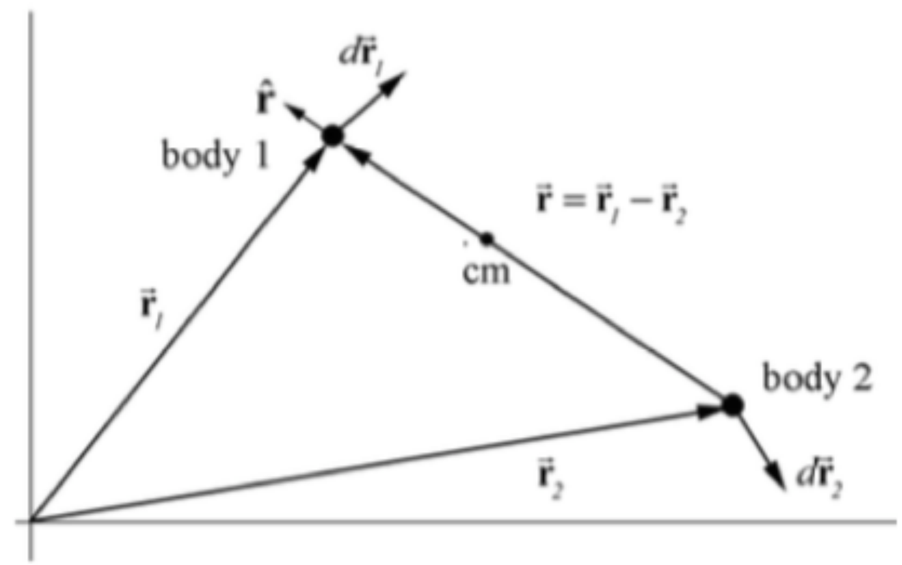

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This simple integration is called a **Euler's Scheme** and gives errors on order of ~ $(\Delta t)^2$ (first order scheme)

** Open up sudo code for Euler’s

* A few notes on Inquiry before we get started…

* Download code and go to it!